

April 8, 2011

## NONLINEAR OPTIMIZATION — Assignment 5

Score composition:

(9.12)	(10.5)	(10.8)	(11.1)	(11.2)	(11.3)	(11.4)	Total
5	5	5	5	5	5	5	35

**(9.12)** Notice that in this question,  $\nabla^2 f$  (or  $H$ ) is not necessarily singular. Though most of you point out that  $\beta$  is the Lagrangian multiplier, to give a strict proof, you should consider the following different cases:  $H$  is singular or  $H$  is invertible; the norm of the unconstrained minimizer is greater than or not greater than  $\gamma$ .

**(11.2)** The associated centering problem

$$\min tx_2 - \log(x_2 - x_1) - \log x_2$$

is actually unbounded below as  $x_1 \rightarrow -\infty$ .

**(11.3)** To solve this question, you may first assume that the sublevel set of the centering problem is unbounded. Then a point  $x$  and a direction  $v$  can be found in this set such that  $x + sv$  are in the sublevel set for  $\forall s \geq 0$ . Since the sublevel set of the original problem is bounded,  $f_0(x + sv)$  is increasing for  $s$  sufficiently large. Therefore, we may choose  $x$  such that  $\nabla f(x)^T v > 0$ . Contradiction then can be found by some simple calculation.

**(11.4)** Because  $x^T x \leq R^2$ ,

$$\frac{1}{R^2 - x^T x} I \succeq \frac{1}{R^2} I$$