Numerical perturbation analysis example: Consider the quadratic program

$$\begin{array}{ll} \text{min} & x_1^2 + 2x_2^2 - x_1x_2 - x_1 \\ \text{subject to} & x_1 + 2x_2 \le u_1 \\ & x_1 - 4x_2 \le u_2 \\ & 5x_1 + 76x_2 \le 1, \end{array}$$

with variables x_1, x_2, and parameters u_1, u_2.

1. Solve this QP, for parameter values $u_1 = -2, u_2 = -3$, to find optimal primal variable values x_1^* and x_2^*, and optimal dual variable values λ_1^*, λ_2^* and λ_3^*. Let p^* denote the optimal objective value. Verify that the KKT conditions hold for the optimal primal and dual variables you found (within reasonable numerical accuracy).
Hint: See Section 3.6 of the CVX users' guide to find out how to retrieve optimal dual variables. To specify the quadratic objective, use quad_form().

2. We will now solve some perturbed versions of the QP, with

$$u_1 = -2 + \delta_1, \quad u_2 = -3 + \delta_2,$$

where δ_1 and δ_2 each take values from $\{-0.1, 0, 0.1\}$. (There are a total of nine such combinations, including the original problem with $\delta_1 = \delta_2 = 0$.) For each combination of δ_1 and δ_2, make a prediction p^*_{pred} of the optimal value of the perturbed QP, and compare it to p^*_{exact}, the exact optimal value of the perturbed QP (obtained by solving the perturbed QP). Put your results in the two righthand columns in a table with the form shown below. Check that the inequality $p^*_{\text{pred}} \le p^*_{\text{exact}}$ holds.

δ_1	δ_1	p^*_{pred}	p^*_{exact}
0	0		
0	-0.1		
0	0.1		
-0.1	0		
-0.1	-0.1		
-0.1	0.1		
0.1	0		
0.1	-0.1		
0.1	0.1		