CO749 **Assignment 1** **Due: September 28**

Problem 1: Prove that a graph G has a K_n-minor if and only if there exist vertex-disjoint trees $T_1, \ldots, T_n$ in G such that, for each $1 \leq i < j \leq n$, there is an edge with an end in T_i and an end in T_j.

Problem 2: Prove that a graph G has tree-width at most k if and only if G can be obtained from graphs with at most $k+1$ vertices by clique-sums.

Problem 3: Prove that, if G is embedded in $\Sigma(h, c; \ell)$ with all of the vertices in the boundary, then G has tree-width at most $f(h, c, \ell)$ (where $f(h, c, \ell)$ is any function of your choosing).

Problem 4:

(a) Prove that, if G has no path of length k, then G has a tree-decomposition of width at most k^2 and diameter at most $2k$.

(b) Prove that, if G has a tree-decomposition of width at most $\omega + 1$ and diameter at most $2d$, then G has no path of length $f(\omega, d)$ (where $f(\omega, d)$ is any function of your choosing).