CO749 Assignment 3 Due: November 2

Problem 1: Let $t, d, m \in \mathbf{Z}_+$ where $d = d(t)$ and $m = m(t)$ are integers of your choosing. Now let G be a graph with a spanning grid subgraph H and let $X = E(G[\delta_d(H)])\backslash E(H[\delta_d(H)])$. Prove that, if G has no K_t-minor, then there is a set $Z \subseteq V(G)$ with $|Z| \leq m$ such that $G\backslash X\backslash Z$ is planar.

Problem 2: Let C be a cycle of length at least 4 in a simple 3-connected graph G. Prove that there exist distinct vertices $x, y \in V(C)$, non-adjacent in C, and a chordless (x, y)-path P such that $P\backslash\{x, y\}$ is disjoint from C and either

- there is a path in $G\backslash V(P)$ connecting the two components of $C\backslash\{x, y\}$, or
- $G\backslash V(P)$ has exactly two components.