Introduction to Quantum Information Processing
QIC 710 / CS 768 / PH 767 / CO 681 / AM 871

Lecture 21 (2017)

Jon Yard
QNC 3126
jyard@uwaterloo.ca
http://math.uwaterloo.ca/~jyard/qic710

Next Tuesday’s Class in QNC1201

Entanglement measures

Entanglement measures

		Monogamous	**Additive**	**Faithful**
Entanglement of formation	E_F	👎	👎	👍
Entanglement cost	E_C	👎	?	👍
1-way Distillable entanglement	$E_D^{\rightarrow}$	👍	👎	👎
Distillable entanglement	E_D	👎	?	👎
Squashed entanglement	E_{sq}	👍	👍	👍

Symmetric extensions

Symmetric extensions

Let $\rho^{AB} = \sum_x p(x)\rho_x^A \otimes \rho_x^B$ be a separable state and consider the state

$$\rho^{AB_1B_2\cdots B_n} = \sum_x p(x)\rho_x^A \otimes \rho_x^{B_1} \otimes \rho_x^{B_2} \otimes \cdots \otimes \rho_x^{B_n},$$

Where $\rho_x^{B_i} = \rho_x^B$ for every i.

What symmetries does $\rho^{AB_1B_2\cdots B_n}$ have?

A general state $\rho^{AB_1B_2\cdots B_n}$ symmetric under interchanging the B_i systems, and for which

$$\rho^{AB} = Tr_{B_2B_3\cdots B_n}\rho^{AB_1B_2\cdots B_n}$$

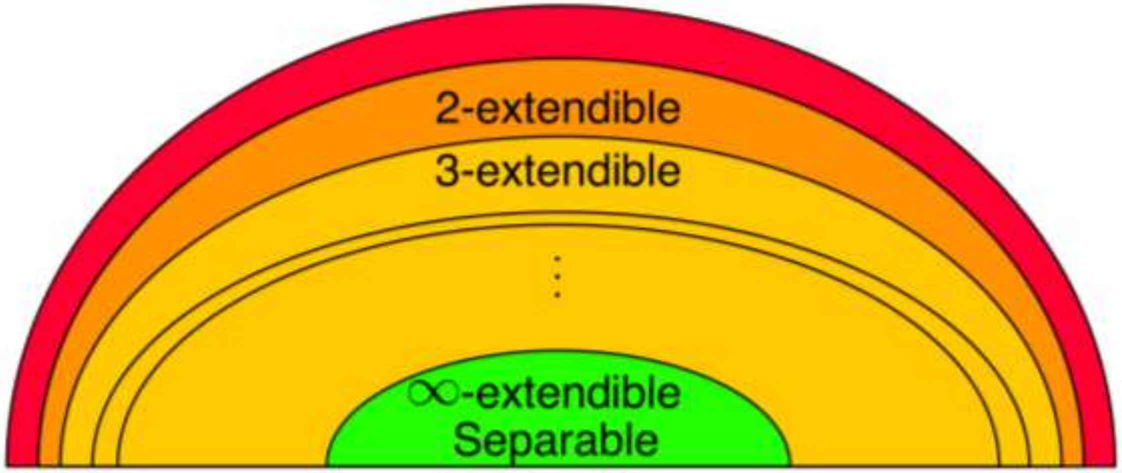
is called a **symmetric $\boldsymbol{n}$-extension** of ρ^{AB}. The state ρ^{AB} is called **$\boldsymbol{n}$-extendible** if it has a symmetric n-extension.

Symmetric extensions

Theorem [Brandao, Christandl, Y'11]:
If ρ^{AB} is n-extendible, then

$$\left\|\rho^{AB} - \mathrm{SEP}\right\|_2 := \min_{\sigma^{AB} \in \mathrm{SEP}} \left\|\rho^{AB} - \sigma^{AB}\right\|_2 \leq \sqrt{\frac{O(\log a)}{n}}.$$

Corollary: ρ^{AB} is separable iff it is n-extendible for all n.



Symmetric extensions

Proof sketch that if ρ^{AB} is n-extendible, then

$$\left\|\rho^{AB} - \mathrm{SEP}\right\|_2 \leq \sqrt{\frac{O(\log a)}{n}}.$$

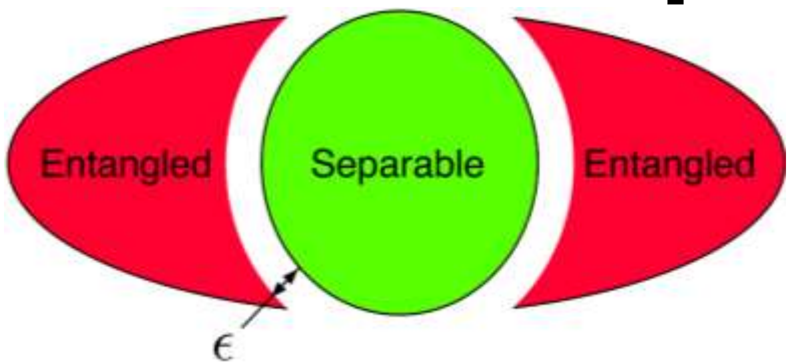
Can show: $I(A;B|C) \geq O\left(\left\|\rho^{AB} - \mathrm{SEP}\right\|_2^2\right)$.

Therefore $E_{sq}(\rho^{AB}) \geq O\left(\left\|\rho^{AB} - \mathrm{SEP}\right\|_2^2\right)$.

But E_{sq} is monogamous and is bounded above by $S(A)$, so

$$\log(a) \geq E_{sq}(\rho^{AB_1B_2\cdots B_n}) \geq \sum_i E_{sq}(\rho^{AB_i}) \geq O\left(n \left\|\rho^{AB} - \mathrm{SEP}\right\|_2^2 \right)$$

Algorithm for separability



Suppose ρ^{AB} is either separable or is ϵ-far from separable.
Then separability can be decided in $\exp\left(O\left(\frac{\log(a)\log(b)}{\epsilon^2}\right)\right)$ time.

Proof (sketch):
If ρ^{AB} is ϵ-far from separable, it not have an n-extension, where $n = \left\lceil O\left(\frac{\log(a)}{\epsilon^2}\right)\right\rceil$. This can be checked by a semidefinite program on $m = ab^n$-dimensional matrices with $M = a^2 b^{2n}$ variables, which can be done in time

$$O(m^2 M^2) = O(a^3 b^{3n}) = \exp\left(O\left(\frac{\log(a)\log(b)}{\epsilon^2}\right)\right).$$