PMATH 965: Topics in Geometry and Topology
Harmonic Maps (WINTER 2025)

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- **Office Hours:** Fridays, 2:00pm–3:00pm
- **Lecture Room:** MC 5417
- **Lecture Times:** 2:30pm–3:50pm

Course description: We study aspects of the theory of harmonic maps. Such maps generalize two familiar objects in Riemannian geometry: real-valued harmonic functions and geodesics. They are also intimately related to the calculus of variations, minimal submanifolds, conformal geometry, and complex geometry. Moreover, the *harmonic map heat flow* introduced in the 1960s by Eells–Sampson was the first example of a *geometric flow*.

Prerequisites: Students should be thoroughly familiar with smooth manifold theory as covered in PMATH 465/665. Some prior exposure to the basics of Riemannian geometry, including Riemannian metrics, the Levi-Civita connection, Riemann curvature, and Riemannian geodesics is helpful but not absolutely essential. We will briefly review these basics at the start of the course.

Textbook: There is no required text. I will be preparing my lectures mostly from the following sources:

- J. Eells and L. Lemaire, "Selected topics in harmonic maps", *CBMS Regional Conf. Ser. in Math.* **50**. Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1983.
- H. Urakawa, "Calculus of variations and harmonic maps", Translated from the 1990 Japanese original by the author. *Transl. Math. Monogr.* **132**. American Mathematical Society, Providence, RI, 1993.

Other useful references are:

- J. Eells and J.H. Sampson, "Harmonic mappings of Riemannian manifolds", *Amer. J. Math.* **86** (1964), 109–160.
- J. Eells and L. Lemaire, "A report on harmonic maps", *Bull. London Math. Soc.* **10**. (1978), 1–68.
- J. Eells and L. Lemaire, "Another report on harmonic maps", *Bull. London Math. Soc.* **20** (1988), 385–524.
- P. Baird, Paul and J.C. Wood, "Harmonic morphisms between Riemannian manifolds", *London Math. Soc. Monogr. (N.S.)* **29**. The Clarendon Press, Oxford University Press, Oxford, 2003.

Brief outline of course topics. (Tentative and definitely subject to change.)

[1] Review of the basics of Riemannian geometry: metrics, Levi-Civita connection, geodesics, curvature.
[2] Riemannian immersions and the second fundamental form. Riemmanian submersions and the O'Neill tensors.
[3] The Laplacian and harmonic functions.
[4] Introduction to harmonic maps, relation to conformal geometry and minimal submanifolds, examples.
[5] Variational formulation of harmonic maps. The first and second variation formulas.
[6] Harmonic maps in complex and Kähler geometry. The theorem of Siu–Yau.
[7] The harmonic map heat flow and the Eells–Sampson theorem.
[8] If time permits: introduction to the existence results of Sachs–Uhlenbeck.

Marking scheme

Course marks will *probably* be determined as follows.

- Assignments: 80% (four assignments, worth 20% each)
- Paper and presentation: 20%

If the course enrolment is very high, I reserve the right to change the marking scheme to:

- Assignments: 100% (five assignments, worth 20% each).

Please note that you are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems.

NOTE: For information on academic offences and accessibility services, please see the detailed version of the course outline available at: https://outline.uwaterloo.ca/view/nnn6c6