PMATH 965: Topics in Geometry and Topology
Harmonic Maps (WINTER 2025)

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- **Office Hours:** TBD
- **Lecture Room:** TBD
- **Lecture Times:** TBD

Course description: We study aspects of the theory of harmonic maps. Such maps generalize two familiar objects in Riemannian geometry: real-valued harmonic functions and geodesics. They are also intimately related to the calculus of variations, minimal submanifolds, conformal geometry, and complex geometry. Moreover, the *harmonic map heat flow* introduced in the 1960s by Eells–Sampson was the first example of a *geometric flow*.

Prerequisites: Students should be thoroughly familiar with smooth manifold theory as covered in PMATH 465/665. Some prior exposure to the basics of Riemannian geometry, including Riemannian metrics, the Levi-Civita connection, Riemann curvature, and Riemannian geodesics is helpful but not absolutely essential. We will briefly review these basics at the start of the course.

Textbook: There is no required text. I will be preparing my lectures mostly from the following sources:

- P. Baird, Paul and J.C. Wood, "Harmonic morphisms between Riemannian manifolds", *London Math. Soc. Monogr. (N.S.)* **29**. The Clarendon Press, Oxford University Press, Oxford, 2003.
- J. Eells and L. Lemaire, "Selected topics in harmonic maps", *CBMS Regional Conf. Ser. in Math.* **50**. Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1983.
- H. Urakawa, "Calculus of variations and harmonic maps", Translated from the 1990 Japanese original by the author. *Transl. Math. Monogr.* **132**. American Mathematical Society, Providence, RI, 1993.

Other useful references are:

- J. Eells and J.H. Sampson, "Harmonic mappings of Riemannian manifolds", *Amer. J. Math.* **86** (1964), 109–160.
- J. Eells and L. Lemaire, "A report on harmonic maps", *Bull. London Math. Soc.* **10**. (1978), 1–68.
- J. Eells and L. Lemaire, "Another report on harmonic maps", *Bull. London Math. Soc.* **20** (1988), 385–524.

Brief outline of course topics. (Tentative and definitely subject to change.)

[1] Review of the basics of Riemannian geometry: metrics, Levi-Civita connection, geodesics, curvature.
[2] Riemannian immersions and the second fundamental form. Riemmanian submersions and the O'Neill tensors.
[3] The Laplacian and harmonic functions.
[4] Introduction to harmonic maps, relation to conformal geometry and minimal submanifolds, examples.
[5] Variational formulation of harmonic maps. The first and second variation formulas.
[6] Harmonic maps in complex and Kähler geometry. The theorem of Siu–Yau.
[7] The harmonic map heat flow and the Eells–Sampson theorem.
[8] If time permits: introduction to the regularity results of Schoen–Uhlenbeck.

Marking scheme

Course marks will be determined as follows.

- Assignments: 75% (three assignments, worth 25% each)
- Paper and presentation: 25%

Please note that you are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems.