Assignment 2

CO 330 (Fall 2026)

Due Friday, October 2, 2026 at 3pm Eastern time.

Instructions

To get full marks on this assignment you must do questions worth 12 marks total. Any additional questions you do beyond 12 marks-worth will not be counted. Each question has two possible point values, one if you would like it marked for correctness and one if you would like it marked for completion and feedback, you will indicate which you want when you submit.

All answers should be justified unless specifically indicated otherwise.

This assignment should be done and submitted individually. You are welcome to discuss with your classmates, but you should not use generative AI for your ideas and you must write up your solutions independently (neither writing it with other people nor with tools such as generative AI). In your write up explicitly acknowledge any discussions or non-class resources and tools that you may have used.

This assignment should be submitted in crowdmark.

Questions

(1) **{4 marks graded for content, 2 marks graded for completion}**
  (a) Let $A_k(x)$ be formal power series for which the partial sums $\sum_{k=0}^{n} A_k(x)$ converge in $R[[x]]$. Prove that $\left(\sum_{n\geq 0} A_n(x)\right)' = \sum_{n\geq 0} A_n'(x)$.
  (b) Prove the chain rule of formal derivatives of formal power series.

(2) **{4 marks graded for content, 2 marks graded for completion}** Determine whether or not the sequence $A_k(x) = \left(1 + \frac{x}{k}\right)^k$ converges in $\mathbb{Q}[[x]]$ as $k \to \infty$. If it does, determine the limit.

(3) **{4 marks graded for content, 2 marks graded for completion}** Let $\mathcal{A}$ be a combinatorial class with no objects of weight 0. Then $\mathsf{MSet}(\mathcal{A})$ is defined to be the combinatorial class of all finite multisets (i.e. repetition is allowed) of objects of $\mathcal{A}$.
  (a) Justify that $\mathsf{MSet}(\mathcal{A})$ is a combinatorial class with the expected weight function.
  (b) Let $\mathcal{B} = \mathsf{MSet}(\mathcal{A})$. Explain why

$$B(x) = \prod_{a\in\mathcal{A}} \left(\frac{1}{1 - x^{w(a)}}\right)$$

  and conclude that $B(x) = \prod_{n\geq 1}(1 - x^n)^{-a_n}$.
  (c) Use properties of formal exp and log to prove that

$$B(x) = \exp\left(\sum_{n\geq 1} \frac{A(x^n)}{n}\right)$$

  This is a good example of how useful combinatorial constructions can be admissible while going beyond sum, product, and composition of series.

(4) **{4 marks graded for content, 2 marks graded for completion}** Prove that the generating series of the class of rooted non-plane (where left and right child are not distinguished) binary trees (still with every vertex having 0 or 2 children) is

$$A(x) = x + x\frac{A(x)^2 + A(x^2)}{2}$$

(5) **{4 marks graded for content, 2 marks graded for completion}** Express each of these combinatorial classes using operations we know ($\mathcal{E}$, $\mathcal{Z}$, other explicit finite sets, $\cup$, $\times$, Seq, etc. You may also use MSet from earlier in this assignment even if you didn't do that question.)

(a) Ternary strings where every block of 2s is followed by at least one 0.
(b) Compositions with an odd number of parts and where each part is even.
(c) Rooted plane trees where each vertex has an even number of children.
(d) Rooted non-plane (order of the children is not distinguished) trees.