Assignment 4

CO 330 (Fall 2026)

Due Friday, November 6, 2026 at 3pm Eastern time.

Instructions

To get full marks on this assignment you must do questions worth 12 marks total. Any additional questions you do beyond 12 marks-worth will not be counted. Each question has two possible point values, one if you would like it marked for correctness and one ("engagement") if you would like it marked for feedback after you have done some work on it. You will indicate which you want when you submit.

All answers should be justified unless specifically indicated otherwise.

This assignment should be done and submitted individually. You are welcome to discuss with your classmates, but you should not use generative AI for your ideas and you must write up your solutions independently (neither writing it with other people nor with tools such as generative AI). In your write up explicitly acknowledge any discussions or non-class resources and tools that you may have used.

This assignment should be submitted in crowdmark.

Questions

(1) **{4 marks graded for content, 2 marks graded for engagement}** Write down the generating series for each of the following sets of partitions with each partition λ contributing $x^{n(\lambda)}y^{k(\lambda)}$ to the generating series.
   (a) Partitions in which only parts divisible by 5 occur.
   (b) Partitions in which odd parts occur at most twice.
   (c) Partitions in which every even part occurs 0 times or an odd number of times and every odd part occurs either 0 or 4 times.
   (d) Partitions for which parts greater than 6 must be distinct.

(2) **{4 marks graded for content, 2 marks graded for engagement}** Let $\mathcal{Y}_{a,b}$ be the set of partitions with first part at most a and with at most b parts. Prove that

$$\sum_{\lambda \in \mathcal{Y}_{a,b}} q^{n(\lambda)} = \begin{bmatrix} a+b \\ a \end{bmatrix}_q$$

(3) **{4 marks graded for content, 2 marks graded for engagement}** Let $\mathrm{pe}(n)$ be the number of partitions of n with an even number of parts, and let $\mathrm{po}(n)$ be the number of partitions of n with an odd number of parts. Let $\mathrm{od}(n)$ be the number of partitions of n which have odd and distinct parts. Show that for all $n \in \mathbb{Z}_{\geq 0}$

$$\mathrm{pe}(n) - \mathrm{po}(n) = (-1)^n \mathrm{od}(n).$$

(4) **{4 marks graded for content, 2 marks graded for engagement}** Make another problem like chapter 9 question 4 or question 5 from `https://melczer.ca/330/WagnerNotes.pdf` or equivalently like question 2 or question 3 of `https://enumeration.ca/extensions/partitions/`, and also write a solution to your problem. That is, find two classes of partitions which are equinumerous by the template

theorem and some algebraic manipulations and justify that this is true. *We can try as a class to find as many different examples as we can.*

(5) **{4 marks graded for content, 2 marks graded for engagement}** The notes by David Wagner, `https://melczer.ca/330/WagnerNotes.pdf`, in chapter 10 contain a proof of the *Euler pentagonal number theorem* via a sign-reversing involution on certain partitions. The proof is 5.5 pages long (with some details left to exercises, but also with a full page example figure included in the proof). Read the proof and summarize the *key ideas* in about half a page. You are not trying to give a re-proof, or necessarily make fully rigorous statements, but you are tasked with communicating what the key ideas are and how they fit together.