Assignment 5

CO 330 (Fall 2026)

Due Friday, November 13, 2026 at 3pm Eastern time.

Instructions

To get full marks on this assignment you must do questions worth 12 marks total. Any additional questions you do beyond 12 marks-worth will not be counted. Each question has two possible point values, one if you would like it marked for correctness and one ("engagement") if you would like it marked for feedback after you have done some work on it. You will indicate which you want when you submit. *Note that this time there is one question that cannot be submitted for engagement.*

All answers should be justified unless specifically indicated otherwise.

This assignment should be done and submitted individually. You are welcome to discuss with your classmates, but you should not use generative AI for your ideas and you must write up your solutions independently (neither writing it with other people nor with tools such as generative AI). In your write up explicitly acknowledge any discussions or non-class resources and tools that you may have used.

This assignment should be submitted in crowdmark.

Questions

(1) **{4 marks graded for content, 2 marks graded for engagement}** Let $\mathcal{A}$ be the set of partitions where each part occurs at most 3 times and let $\mathcal{B}$ be the set of partitions where each even part occurs at most once (and no constraint on odd parts). Showing equinumerosity of $\mathcal{A}$ and $\mathcal{B}$ via generating series is one of the problems of the form of assignment 4 question 4. For this question, give a weight-preserving bijection between these two sets.

(2) **{4 marks graded for content, 2 marks graded for engagement}** Question 7 from `https://enumeration.ca/extensions/partitions/`

(3) **{4 marks graded for content, 0 marks graded for engagement}** *Note that this question cannot be submitted for engagement!* The two sets of notes we are using (`https://enumeration.ca/extensions/labelled-constructions/` and `https://melczer.ca/330/WagnerNotes.pdf` chapter 11), use different notation for operators/constructions on labelled combinatorial classes. Make a dictionary relating the notation between the two sources, that is, for each construction give a translation for what it is in the notation of the one source and in the notation of the other source. If there are slight discrepancies, give the translation that works the bulk of the time but also clarify how the edge cases differ.

(4) **{4 marks graded for content, 2 marks graded for engagement}** Question 1 from `https://enumeration.ca/extensions/labelled-constructions/`

(5) **{4 marks graded for content, 2 marks graded for engagement}**

   (a) Give an example of a combinatorial class built of atoms with the property that for each size there are the same number of labelled objects as unlabelled objects of that size.

(b) Give an example of a combinatorial class built of atoms with the property that for each size n, there are $n!$ times as many labelled objects as unlabelled objects of size n.

(c) Give and example of a combinatorial class where the number of labelled objects compared to unlabelled objects is intermediate between the extremes of the previous parts. Plot the first 100 terms of the counting sequence for the labelled and unlabelled version of this class.