CO 330 LECTURE 14 SUMMARY

FALL 2026

Summary

We looked at q-counting for lattice paths today.

We defined $\mathcal{L}(a,b)$ to be the set of lattice paths from $(0,0)$ to (a,b) using steps $\uparrow$ and $\rightarrow$. We did an example. Then $|\mathcal{L}(a,b)| = \binom{a+b}{a} = \binom{a+b}{b}$, but moreover, letting area(P) be the area under the path P we have

Proposition 1.

$$\sum_{P \in \mathcal{L}(a,b)} q^{area(P)} = \begin{bmatrix} a+b \\ a \end{bmatrix}_q$$

which we proved by using the bijection with subsets of $\{1, \ldots, a+b\}$ where the subset indexes the horizontal steps. Then considering the area under each horizontal step we can use our subset q-counting result from last time. Details can be found in the enumration.ca notes.

I also noted but didn't prove

Proposition 2. *Fix $n, k \in \mathbb{Z}_{\geq 0}$ with $k \leq n$ and let $q = p^c$ be a prime power. Then the number of k-dimensional subspaces of an n-dimensional vector space over $\mathbb{F}_q$ is $\begin{bmatrix} n \\ k \end{bmatrix}_q$ where $\mathbb{F}_q$ is the finite field with q elements.*

This one is on your assigment 3.

Next you worked on three classical binomial identities in groups, first finding a combinatorial proof of the classical version and then finding a q-generalization. The approach you should take here is to take your combinatorial proof, maybe make it more precise if needed, and then just see what the qs do there, using the formulas we know for q-counting permutations, subsets, and lattice paths. This will give you a q-generalization of the binomial identity and a proof of it.

Not all the groups had time to finish, so we'll get back to this on Monday.

Next time

Continuing your q-identities in groups and then on to partitions.

References

- `https://enumeration.ca/extensions/q-analogues/`
- Chapter 5 of `https://melczer.ca/330/WagnerNotes.pdf`