CO 330 LECTURE 23 SUMMARY

FALL 2026

Summary

I did one more labelled example for you. Let $\mathcal{T}$ be the class of labelled trees (as graphs, so not rooted). $\mathcal{T}$ is hard to decompose but $\mathcal{T}^\bullet$ is much easier, so we can find the exponential generating series of $\mathcal{T}^\bullet$ and then use that to understand $\mathcal{T}$. Specifically

$$\mathcal{T}^\bullet \leftrightharpoons \mathcal{Z} * \mathsf{Set}(\mathcal{T}^\bullet)$$

so $T^\bullet(x) = xe^{T^\bullet(x)}$. This is well-suited to LIFT so the number of labelled rooted trees with n vertices is

$$n![x^n]T^\bullet(x) = n!\frac{1}{n}[u^{n-1}]e^{nu} = (n-1)!\frac{n^{n-1}}{(n-1)!} = n^{n-1}$$

So the number of unrooted labelled trees with n vertices is $n^{n-1}/n = n^{n-2}$ which is a very famous formula.

Then you worked on some further examples in groups.

Next time

You will continue working on the group examples and then we'll review for the midterm.

References

- `https://enumeration.ca/extensions/labelled-constructions/`
- Chapter 11 of `https://melczer.ca/330/WagnerNotes.pdf`