CO 431/631 ("Symmetric functions")

Overview

Symmetric functions are at the heart of much of modern algebraic combinatorics. The course will give a broad introduction to the algebra, combinatorics, and applications of symmetric function theory. Major topics will include:

- The algebra of symmetric functions: bases and other algebraic structure
- Combinatorial structure: Schur symmetric functions and Young tableaux
- Applications to combinatorial enumeration, representation theory, and enumerative geometry

Level of the course

CO 431/631 is intended for graduate students and senior undergraduate students who have completed the required algebra courses.

Official prerequisites

For undergraduate students, the following prerequisites are required to take the course.

- PMath 336 or PMath 347
- Overall cumulative average of 80%

Math 235 (or Math 245) is also required background. This is not listed as an official prerequisite, because it's a prerequisite for PMath 336/347. In addition, Math 239 (or Math 249) is recommended. If you took PMath 336 but not PMath 347, it is also recommended that have taken PMath 334. These recommended courses are not official prerequisites, and it is possible to take the course without them, however you should be prepared to ask questions in office hours.

For graduate students, there are no official prerequisites. You are expected to be able to make your own informed decision about whether the course is appropriate for you, after reading the information below.

This rest of document provides details about the background knowledge you will need for CO 431/631. You can use this information to decide if this course is right for you, and to decide if there are any topics you want to review before the course starts. If you have any questions or concerns, please do not hesitate to ask.

Background material: Overview

The main prerequisite areas are group theory, linear algebra, ring theory, and combinatorics. Below, you will find a list of relevant topics for each of these areas. The symbol next to each topic is indicates how much familiarity I expect you to have:

"•" means I expect you to be familiar with the topic; these topics will not necessarily be reviewed in class, and it is your responsibility to make sure you understand this material.

"◦" means I expect you have seen the topic before, but I will most likely give a brief review of it in class.

"—" means it would be great if you know about this, but I expect some students will be unfamiliar with the topic, and that's okay.

Topics marked as "•" or "∘", should be treated as required background for the course. If you are less familiar with some of these, it would be advisable to review/relearn them on your own, preferably before the course begins. If you can do this, you should be well-prepared to take CO 431/631.

For topics marked as "—", I will not assume you know all of this material. In most cases, I will be covering/reviewing this material in class. However, my hope is that not everything here will be new to everyone. If you are familiar with some of this material, it will make the course a little bit easier.

Group Theory

An introductory course in group theory is required background for this course. Group actions are a central theme of the course. It will be very important to be familiar with the basic definitions and theorems from group theory, especially those relating to group actions.

Relevant topics:

- • Definition of a group and examples
- • Group homomorphisms and isomorphisms
- • Subgroups and normal subgroups
- • Conjugacy classes
- ∘ Actions of finite groups on sets
- ∘ Orbits, stabilizers, fixed points
- ∘ The symmetric group
- ∘ Presentation of groups (generators and relations)
- — Orbit counting ("Burnside's lemma")

Linear Algebra

A strong background in linear algebra is probably the most important requirement. Linear algebra will be used extensively in this course, in a variety of different ways.

Relevant topics:

- • Vector spaces over $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$
- • Linear independence, span, bases
- • Linear transformations and operators, and their matrix representations
- • Subspaces, direct sums
- • Characteristic polynomial, eigenvalues and eigenvectors
- • Properties of determinant and trace
- • Inner product spaces (orthonormal bases, unitary operators, etc.)
- ∘ Bilinear maps
- ∘ Invariant subspaces for a linear operator
- — Linear algebra over finite fields

— Free vector spaces, linear and bilinear extensions of functions
— Dual spaces
— Tensor products

Ring Theory

A introductory course in ring theory is recommended background, but not required. A lot of ring theory is more general and more abstract than anything needed for this course. You should be able to get by without a full course in ring theory, as long as you are familiar with a few basic concepts, such as the definitions of a ring, and a ring homomorphism.

Relevant topics:

- • Definition of a ring and examples
- • Ring homomorphisms and isomorphisms
- ◦ Algebras over a field
- ◦ Generators of rings/algebras
- — Algebraic independence
- — Non-commutative rings/algberas

Combinatorics

An introductory course in combinatorics, such as Math 239 is recommended. Since this is an algbraic combinatorics course, we will be using a healthy mix of algebraic and combinatorial tools. You should be familiar with basic concepts like graphs, and generating functions, though you don't necessarily need to know all the theorems and tricks. An enumeration course such as CO 330 would also be beneficial, but not required.

Relevant topics:

- • The ring of Formal Power Series
- • Definition of a graph
- • Basic graph theory concepts (e.g. isomorphisms, cycles, connectedness)
- • Definitions of partially ordered sets, totally ordered sets
- ◦ Ordinary generating functions
- — Integer partitions
- — Directed graphs
- — Exponential generating functions