OPTIMIZATION BASED APPROACHES TO PRODUCT PRICING

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Key words: *Product pricing, optimization, integer programming, convex programming*

—ABSTRACT —

We consider various axioms for customer behaviour using utility functions and so-called "reservation prices" and then based on these axioms, we discuss some mathematical models (employing integer programming, convex programming and classical nonlinear programming) for deciding on product prices to maximize the total profit (or perhaps another suitable objective function also involving minimization of risk). We also share some of our experiences from a recent collaborative research project involving a company in the tourism sector.

JEL Classificiation: C61, C63, L11, L83

1. INTRODUCTION

Consider a company with multiple products in the same market. Let us focus on the sales and revenue for our company. We will direct our attention to pricing our products so that we maximize the total revenue. To begin, let us assume that all the products are ready (we already committed the production costs and the corresponding capacities) so that our current problem is only about maximizing the total revenue which at this stage will maximize the total profit.

We must be very careful about two main issues. The first issue is rather obvious: to make a sale, one of our products must win a customer over all the products of the competing companies. The second one is slightly less obvious: suppose a group of potential customers are going to buy a product of our company, then we must make sure that our prices do not motivate the customers to move from high revenue products to low revenue products. When this second issue is not managed properly, we say that *cannibalization* occurred. In such a case, the lower revenue generating product cannibalizes the higher revenue generating product (or products). Interestingly enough, there are genetic algorithm based optimization approaches that try to weed-out pricing policies with these types of cannibalistic tendencies (see Fruchter et al. (2006)).

Now, let us try to make the problem more general (and more realistic) by including decisions about production levels (and the related capacity allocation). This new level of decision making can usually be accommodated by existing optimization models. Indeed, a key point is to make sure that the part of the optimization model dealing with the pricing decisions is compatible with the existing optimization models which deal with production planning (and capacity constraints). The underlying optimization models and techniques in the literature involve heuristics, dynamic programming approaches, mixed integer programming models and classic nonlinear programming models.

To be able to employ the above optimization machinery, one needs to do a lot of work with the data using data mining techniques. These come into play when we try to model customer behaviour and preferences (in a nut shell, the demand). However, we also need to extract enough information from the decision makers in the company about the preferences of the top management so that we can construct a suitable objective function. In addition, the competitors' products and customers' view of them must also be quantified and incorporated into the optimization model.

In this paper, I will take a high-level view of optimization techniques in product pricing and advocate certain new approaches as promising avenues of further research. In particular, for incorporating the customer preferences, I like the use of *reservation prices.* One of the ways of using the reservation price concept is to segment the potential customers into groups so that the customers within each segment have similar behaviour as far as their preferences and buying power are concerned. Then, for each customer segment, we try to deduce the price that the customers are willing and able to pay for each of our products as well as the products of competing companies. Clearly, the notion of reservation price lies in the general realm of the well-established concept of utility.

In the next section, we discuss ways of modelling customer preferences that are suitable for optimization models. Then in the following four sections we mention various optimization models and approaches. Section 7 is a brief discussion of the computational state-of-the-art. In Section 8, we discuss a few aspects of a recent collaborative research project with a company in the tourism sector. Concluding remarks featuring some future research and application directions are in Section 9.

Understanding the demand for our products and the competing products in the market is always extremely important. How do we quantify the customer behaviour in a useful, robust way? This is the question we address next.

2. CUSTOMER BEHAVIOUR

The first question is: "how many potential customers do we have?" In some settings this number may be extremely large (e.g., on the order of a hundred thousand or even millions). One effective approach is to create customer segments. That is, we group the customers so that within each group, the customers have strong common characteristics about their preferences and financial power (this implies that customers within each segment behave essentially like a single customer) and across different groups (segments) there are some significant differences and distinguishing features.

The next step can be the determination of quantitative techniques that will translate the raw data about these customer segments into computable quantities. One general approach is to determine a distribution of the demand for our products from the above mentioned data. A common implementation is to assume a normal distribution (with certain mean and variance) for each customer segment. The mean and the variance for the distribution would be computed from the raw data for each segment and/or it could be tempered with, using the historical expert knowledge from the marketing/sales department of the company. In such an implementation, the optimizer samples from the demand distribution and then applies a dynamic programming or a Monte-Carlo integration technique. For detailed descriptions of such fundamental approaches and the most commonly used heuristic algorithms, see the comprehensive book (van Ryzin and Talluri (2004)).

Note that it is unclear whether there is a probability distribution that governs the customer preferences (in fact the most likely answer is "no"). So, the probabilistic approach above is really meant to deal with the fact that the information available to the company is far from perfect. Therefore, the quantification of customer segmentation, customer preferences and customers' financial capacity is a very rough procedure, only leading to rough estimates. By randomly sampling from a "representative" probability distribution (determined by these rough estimates), we attempt to protect our approach from yielding irrelevant and misleading results.

Now, we are faced with the final step in determining the customer behaviour: How do we decide whether a customer or a customer segment buys a specific product in the market? If we assume an underlying utility function for each customer segment, under the axiom that a customer will buy the product which maximizes his/her utility, we can arrive at some concrete mathematical models which in turn lead to optimization problems. To make the discussion more specific, let n denote the number of products of our company, name the products 1, 2, . . . , n and let π_j denote the price of our product j (note that π_j is a decision variable in our context). Since we have our customer segments, let us name them similarly: 1, 2, . . . ,m. We denote by R_{ij} the reservation price of customer segment i for product j. I believe that if we employ an approach which treats R_{ij} as data, then we must be mindful of the fact that we simply accepted the following as an axiom:

Axiom 0. R_{ij} the reservation price of customer segment i for product j (or a reasonable approximation of it), is available to the company for every customer segment i and every product j.

We will consider the applications where a certain simple principle of fairness is observed.

Axiom 1. For each product, every customer pays the same price.

The next axiom allows for simplifications in the theory. Moreover, it is not very restrictive (there are heuristic ways of modifying the optimization models to deal with the general case; usually replicating a customer many times---let us call the copies of the customer clones---and making suitable changes to the R_{ij} values of the clones of the customer can be satisfactory).

Axiom 2. Each customer buys at most one product.

Now, we can define the surplus based on the reservation prices and the unknown decision variable as the difference (R_{ij} - π_j). It is reasonable to assume that customer segment i will not buy product j if the corresponding surplus is negative.

Axiom 3. No product with negative surplus is bought.

Thus, only (customer segment, product) pairs that are in-play are those (i,j) with (R_{ij} - π_j) $\geq$ 0.

Our next step will cause multiple branches in our reasoning. So, we will name the axioms accordingly:

Axiom 4.a. (Maximum Surplus) If customer segment i has nonnegative surplus for some products then the segment will buy the product with the largest surplus.

Note that Axiom 4.a. may need a tie-breaking rule if for a customer segment i, there are multiple products which attain the maximum nonnegative surplus. Shioda, Tunçel and Myklebust (2007) propose a tie-breaking rule based on a parameter called *utility tolerance.* This tolerance is a positive amount by which the surplus of the winning product must beat all the other competing products. One consequence of this rule is that there are prices that the company may choose for which the surplus may be positive but the utility tolerance tie-breaking rule may not be met (this would imply that the customer segment would buy none of the products). This in turn results in a pessimistic (but more robust) estimation of the total potential revenue. However, not having such a tie-breaking rule could result in the assumption that the customer segment i buys the most expensive product among all products with maximum positive surplus, perhaps yielding to gross over-estimation of the total potential revenue as well as sales.

Axiom 4.b. (Maximum Utility) For each customer segment i, there is a utility function $u_i(.)$ which determines (as a function of the surplus) the probability that customer segment i buys product j. Moreover, the function $u_i(.)$ is monotone nondecreasing.

In terms of generality, Axiom 4.b. is clearly at the more abstract end of the spectrum in relation to Axiom 4.a. The subject of utility theory, in the context of customer behaviour, is well-studied in mathematical economics and there exist many interesting choices for utility functions. However, we are interested in large-scale revenue management problems and for such problems, maximization of complicated nonlinear functions might not allow for obtaining near-optimal solutions (prices) with reasonable computational resources. This motivates the next axiom (which is an over simplification).

Axiom 4.c. (Uniform Buyer) Probability that the customer segment i buys product j is uniformly distributed among all products for which the customer segment has nonnegative surplus (this probability is zero for all products for which surplus is negative).

We said Axiom 4.c. was an over-simplification, because if the customer segment i has only two products, say 1 and 2, with surplus 0 and 100 respectively, according to Axiom 4.b., the customer segment i will buy product 1 with probability half (and the same probability for product 2). Clearly, this axiom is almost impossible to justify. However, Shioda, Tunçel and Hui (2007) report computational experiments (with the same randomly generated data or with data from applications) indicating that actual optimal prices delivered by a revenue optimization model operating under Axiom 4.c are generally very close to the optimal prices delivered by a revenue optimization model operating under Axiom 4.d. below:

Axiom 4.d. (Share-of-Surplus) Probability that the customer segment i buys product j is the ratio of the surplus for product j to the sum of positive surpluses for customer i over all products. (This probability is zero for all products for which surplus is nonpositive).

For example, if customer i has surpluses 10, 90 and 100 for products 1, 2, 3 respectively, then the probability that customer i buys product j is 0.05, 0.45, 0.50 respectively (see for instance, Kraus and Yano (2003)). Share-of Surplus axiom seems quite reasonable; however, it has at least two drawbacks:

- The optimal prices obtained under this axiom are sensitive to the number of competing products with substantial surpluses.

- The resulting optimization problem is hard to solve (especially for large number of customer segments and products).

For an optimization model related to Axioms 4.c. and 4.d. see the weighted-uniform model of Shioda, Tunçel and Hui (2007).

Now, let us turn to the axioms relating to the competition from other companies. Perhaps one of the strongest is:

Axiom 5.a. (Static Competition) Competitors' prices are fixed for the duration of the planning horizon and are accessible to us.

This axiom, while very unreasonable looking, makes the optimization models much easier to state and solve. Moreover, if we keep our planning horizon very short (say we monitor the market daily and re-optimize our prices every night), then the underlying optimization models (with the unreasonable Axiom 5.a.) might still give a workable set of optimal prices. Another approach might be to build the optimization model conservatively so that optimal prices delivered by the optimization model are robust under small perturbations of competitors' prices.

Axiom 5.b. (Game Theory in the main market as well as in the closely related markets) Competitors' prices are dynamic and are influenced not only by our price changes but also by the changes in the other markets with significant input-output relations to our market. All competitors aim to maximize their total profit in a four year planning horizon, where the principles of basic game theory govern their tactical moves.

The last axiom gets closer to modelling reality; nevertheless, it is far from complete. The basic principles of game theory require further axioms about whether some of the companies are cooperating and to what degree. Some of the competing companies (or our company) may be a part of a holding which includes main players in a closely related market with very large input-output coefficients in relation to the market under study (we do not even attempt to get into discussing the possibility of the companies in the market trying to create an oligopy or government intervention in the market which in turn can make non-price based competition a very important factor in the market). All these factors, if directly included in the model, would make the revenue management problem unmanageable! We do not advocate ignoring these important aspects of the problem, but rather recommend that these aspects be dealt by the "art" side of operations research techniques instead of the very concrete and computationally well-established "optimization machinery." The main problem can be decomposed, those individual pieces amenable to computational optimization techniques can be dealt with and then the individual "optimal policies" can be pasted together by using an artful form of an optimization approach that is relatively robust to possible small changes in the circumstances of the subproblems.

3. GENERAL APPROACH TO OPTIMIZATION PROBLEMS

Some of the traditional approaches for revenue management tend to use stochastic dynamic programming techniques. In this paper, we have been gearing our axioms and models towards the techniques of mixed integer programming and modern convex relaxation approaches. Maximum utility based pricing models in particular seem very amenable to mixed integer programming techniques. Nevertheless, some recent work (see Dussault et al. (2006)) used a bi-level pricing model. In this setting, one relaxes Axiom 5.a. and treats our company as the "leader" and accounts for reactions of the competitors to our prices (this approach has good potential in oligopy markets). In the absence of favourable special structure, the underlying optimization problems become very hard indeed. Dussault et al. (2006) use heuristics and ideas from interior-point methods to compute good prices in a reasonable computation time. The number of products in their computational experiments is not in the order of millions but is bounded by 500.

It seems that we have discussed too many axioms already. However, to make our approach more widely applicable, we need to expand on the question "what makes a product" some more. In many interesting applications of revenue management, optimal product pricing problem is intertwined with the optimal bundling problem. Companies offer products singly and/or in bundles. For example, consider a company in the tourism sector. The company offers 2-day weekend get-away packages (with airline tickets, hotel, car rental), 5-day, 7-day, 14-day vacation packages, 5-day, 7-day, 9-day cruises (with airline tickets, cruise tickets, excursions included), etc. As we drill down further into the details, the number of product bundles increase very fast (very modest numbers such as 6 type of airline tickets, 24 type hotel rooms, 5 options for car rentals, 12 vacation locations, 10 potential departure cities, 4 different departure days, 4 different return days lead to more than 5.2 million product bundles, a number which does not even include the cruise option or other travel options). Thus, we see that a detailed optimization model should be able to deal with millions of variables (since the bundles in the above example, which are in the millions, are represented by "products" in our axiomatic revenue management models). In such applications, we can have the number of customer segments on the order of thousands or tens of thousands. It is possible to write mixed integer programming models which deal with the exponentially many product bundles implicitly (see Hanson and Martin (1990)); however, these mixed integer programs are very hard to solve since their linear programming relaxations provide very poor approximations to the convex hull of feasible solutions of the original mixed integer programming problem.

On the positive side, there was already some encouraging news in the early 1990's as the heuristic algorithm of Dobson and Kalish (1988, 1990) arrived at the scene. This heuristic algorithm has been implemented to run very fast in practice and usually delivers prices that yield close to the maximum revenue. The main subroutine of Dobson-Kalish heuristic solves a shortest path problem in a directed network (without a negative cost cycle), where the number of nodes is bounded by the number of products n. Together with such heuristic algorithms (that are fast in practice), mixed integer programming approach becomes more promising with the introduction of strong valid cuts which significantly strengthen the linear programming relaxations and allow fathoming of a very large number of subproblems in the branch-and-bound algorithms applied. For some of the new cuts, see Shioda, Tunçel, Myklebust (2007).

4. MIXED INTEGER PROGRAMMING MODELS

Under Axioms 0., 1., 2., 3., 4.a. (or 4.c.), and 5.a., defining a binary decision variable θ_{ij} which takes the value 1 when customer segment i buys product j and takes the value 0 otherwise, allows formulating the revenue management problem as a mixed integer programming problem with continuous decision variables π_j (representing the prices to be decided) and 0,1 variables θ_{ij} (representing the assignment of the customer segments to products under the axioms and based on the prices π_j). We think of θ_{ij} and prices π_j as the main variables of the formulations, but typically some additional auxiliary variables are also needed. Note that the number of binary variables is at least mn.

5. NONLINEAR, NONCONVEX FORMULATIONS

Now, let us assume Axioms 0., 1., 2., 3., 4.d., and 5.a. This allows formulating the revenue management problem as a nonlinear programming problem. We can easily reformulate the problem to make the objective function linear, but this transformation leaves the feasible region highly nonconvex. Moreover, finding tractable convex relaxations which provide very good approximations for these nonconvex formulations seem very hard. For computational experience with such nonlinear optimization problems in small scale, see Kraus and Yano (2003).

Some of the current best formulations put the problem into a mixed-integer fractional programming problem with linear constraints. In these formulations, the number of 0,1 variables is mn and the objective function is nonconvex. Even if we consider relaxing the integrality condition on the 0,1 variables, the underlying relaxation still seems intractable for large or even moderate scale problems (even though the feasible region of the relaxation is a polyhedron, the objective function is still nonconvex). Despite the above-mentioned difficulties, this approach still holds significant promise.

6. CONVEX RELAXATIONS OF NONLINEAR, NONCONVEX FORMULATIONS

The previous section motivates reformulations of these problems as mixed 0,1 nonlinear programming problems with the property that upon relaxing 0,1 variables to take any value on [0,1], the relaxation becomes convex. These approaches allow the employment of modern second order cone programming and semidefinite programming techniques. While the latter techniques do not behave as well as linear programming when incorporated into a branch-and-bound or branch-and-cut scheme, they have the potential of providing much stronger relaxations resulting in stronger bounds and fathoming a large number of subproblems.

7. COMPUTATIONAL STATE-OF-THE-ART

For most of the computational work on the models based on our axioms, the number of products and the number of customer segments both seem to be bounded by 500. This even includes heuristic algorithms with no guarantees in terms of proximity to the optimal value. On the side of extremely large-scale problems, Shioda, Tunçel, Myklebust (2007) solve some mixed integer programming problems coming from a class of maximum utility revenue management problem with about 100 million variables and about 800 million constraints in roughly one hour (on a fast computer circa 2006) to within 6% provable optimality. Typically such solutions (the underlying revenue/pricing strategy to be implemented by the company) are obtained within an hour of computation time. However, proving computationally that the generated solution is near-optimal requires about a week of CPU time.

When we discussed Axiom 5.a. (Static Competition), we mentioned that one remedy (to relax the axiom) would be to adjust our optimal prices to the changes in the market by periodically reviewing them (e.g., daily). To make our optimization approach with 800 million variables viable in applications, we should be able to very quickly re-optimize the prices in the face of small changes in the market. Shioda et al. (2007) also report on such experiments. Their implementation of the algorithms was able to re-optimize within a matter of minutes when there were moderate changes (10 to 20 percent) in the prices in the market and in the preferences of the customers.

8. ON THE TOURISM SECTOR APPLICATIONS

There are very large bodies of existing work in the areas of "airline revenue management" and "hotel revenue management." (See, for instance, McGill and van Ryzin (1999), van Ryzin and Talluri (2004) and Siguaw and Kimes (2003).) However, combination of these two components with the addition of car rental, cruise, etc. options generate new challenges. One of my recent consulting experiences was with a company in the tourism sector which sold vacation packages. The company was a branch of a parent company which is a major airline. The companies which sell vacation packages in the tourism sector are sometimes called "tour operators." Tour operators' revenue management problem is essentially packaging the components: airline tickets, hotel rooms, car rentals, cruises, excursions, etc. Each company may present a different situation than the average tour operator.

For example, if the tour operator has a very close relationship to a major airline, the nature of this relationship can have a profound effect on what the objective function of the revenue management problem should be. However, the general structure of the optimization problem mostly stays the same and the computation times for the solution techniques do not seem to be negatively affected.

An important complication with one of our axioms arises in the airline industry (where this version of revenue management was born). One of the pioneers in the industry has been American Airlines (AA). Since the late 1970's, low-cost airlines kept penetrating into the airline market in a substantial way buy cutting prices and providing direct flights to small airports. Major airlines quickly realized that they could not win a price war against these low-cost companies. One response by AA was to create purchase restrictions, booking limits and capacity controlled fares. This response now is widely used as a main pricing strategy for essentially every major airline in the World. For our purposes, we must notice that this pricing policy does not seem to obey Axiom 1. (Different customers on the same flight pay different amounts for essentially the same type of seat and service.) However, all is not lost. From a mathematical standpoint, we simply define a different product for each of these price incentives (e.g., buy ticket at least one month in advance, sleep over on a Saturday). Therefore, even though our Axiom 1. looks very restrictive, it does not mathematically exclude the treatment of the above pricing situation seen in the major airlines.

So far, we have not discussed the capacity constraints. One reason is that we are able to incorporate them to all mathematical models that we discussed with relative ease. Moreover, we are also able to deal with different "types" of capacity. For instance, many tour operators reserve blocks of hotel rooms in various vacation destinations. These reservations are done with respect to different agreements and can contain different sets of cancellation rules (and penalties). Optimization approaches have a positive role to play here as well. On the one hand, if these agreements have already been signed then they should lead to concrete cost analysis. Since our objective function is in terms of revenue, costs and capacities can be easily incorporated to our optimization models (without making the optimization problem intractable). On the other hand, having the optimization model and data ready allows the company to better negotiate these hotel reservation agreements with the suppliers (for the next planning horizon) by using what-if scenerio analyses on the latest data and forecasts.

9. CONCLUSION

One of the most important challenges of revenue management problems that we considered is the lack of knowledge about the data. We can only hope to get a rough estimate of what the reservation prices are for our potential customers. So, whatever optimization model/technique combination is used, it must be designed to be robust with respect to the changes in the data. In particular, the optimal or near-optimal prices delivered by the optimization techniques should be robust under modest perturbations of the reservation prices of the potential customers and the competitors' prices.

In optimization theory, there are areas such as sensitivity analysis, stochastic programming, chance-constrained programming and (more recently) robust optimization each of which deals with the lack of knowledge in the data. Among these,

- multi-stage recourse models in a stochastic optimization framework and
- robust optimization

are very promising for product pricing problems.

With the World-Wide-Web sales becoming the main interface between the companies and the consumer, the emergence of the optimal pricing tools as discussed in the current paper makes it very tempting to directly connect the optimal price computing algorithms to the websites that the customers use. While it is very useful to receive and record all possible website activity by the potential customers, it is indeed very dangerous for companies to blindly rely on the optimizing software to directly respond to the changes via the WWW. Every pricing change suggested by the optimization software must be watched and carefully scrutinized by an experienced (human) revenue manager before any final pricing decisions are made.

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