An improved interior-point approach for use in reservoir operation

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ABSTRACT. A novel approach is presented for solving multi-reservoir operation planning (MROP) problems. MROP is formulated as a multi-objective optimization model and solved by a sequential linear programming (SLP) technique. The resulting linear programming (LP) problem is often very large and highly sparse. A constant-potential interior-point algorithm is developed for handling LP problems in general. The efficiency of the algorithm is further improved by exploiting the structure of MROP. In particular, the normal equation arising from the projection step of the algorithm is solved more effectively, given the sparsity structure of this kind of LP problem. A good initial solution is also introduced from which the algorithm converges to an optimal solution in much less computational time. Finally, some experimental results demonstrate the fine performance capabilities of the algorithm.

1 INTRODUCTION

Determining the optimal operation of a multi-reservoir system is a complex problem that confronts decision makers on a regular basis. The complexity of the system mainly stems from interactions among several reservoirs located in series and parallel, the existence of conflicting objectives, and the stochastic nature of the system.

Most reservoir systems are operated to serve multiple objectives. Based on US Army Corps of Engineers records, about 56% of the storage capacities in the United States are designated for multiple-purpose uses (Wurbs, 1991). Reservoir purposes include flood control, hydropower generation, water supply, water quality enhancement, navigation and recreation. The usefulness of the model relies on how well the multi-objective characteristic of the system is reflected in the model. The primary purposes are usually dealt with as individual objectives to be optimized by the model. The secondary goals may be treated as fixed constraints to be satisfied at certain levels. Many multi-objective techniques have been proposed and applied to reservoir operation problem including Cohon and Marks (1975) and Ko et al. (1992). Ko (1989) suggests that the ϵ-constraint method is the preferable approach. This method places all the objectives but one into the constraints, subject to some specified bounds.

Successive linear programming (SLP) has been extensively used as a viable approach in water resources applications (Yeh, 1985; Loucks et al., 1981). Grygier and Stedinger (1985) have compared the performance of SLP with other optimization techniques, based on experimenting with three different reservoir systems. The authors conclude that SLP dominates the other methods in terms of efficiency and global convergence. Soliman and Christensen (1986) propose a stochastic SLP approach for the optimization of long-term operation. Reznicek and Simonovic (1990) employ a similar model in their new algorithm for hydropower optimization, named EMSLP.

SLP has become even more desirable because of new advances recently made in solving linear

programming (LP) problems. Interior-point (IP) methods have proven to be extremely efficient in solving very large LP problems. The primary purpose of this research is to develop a tailor-made IP algorithm for multi-reservoir operation planning (MROP) problems. A new constant-potential IP algorithm is developed and further enhanced by taking the sparsity structure of MROP into account. Ponnambalam et al. (1989) applied a general dual-affine IP algorithm to a system of two reservoirs in series. However, in this research, it is intended to exploit the problem structure in order to design a special IP algorithm. The authors believe that the high capability of IP methods can effectively handle the dimensionality problem associated with this application.

In the next section, the mathematical formulation of an MROP problem is given. In Section 3, after a brief review of previously developed IP methods, the solution methodology as well as the proposed algorithm are described. Section 4 focuses on exploiting the MROP structure while Section 5 introduces a good initial solution for the proposed algorithm. Section 6 presents some numerical experiments and finally, some conclusions are drawn in Section 7.

2 PROBLEM FORMULATION

The dynamics of a K-reservoir system are mathematically described as:

$$\mathbf{S(t+1) = S(t) - F\ R(t) - D(t) - L(t) + I(t)}, \quad (1)$$

where $\mathbf{S}(t)$ and $\mathbf{R}(t)$ represent vectors of storage variables and turbine release variables at the period t, respectively. $\mathbf{D}(t)$ denotes the vector of direct releases for water supply. The $(K \times K)$ coefficient matrix of $\mathbf{F}$ is specified by the reservoir system configuration. Its diagonal elements, f_{kk}, are all equal to 1. The off-diagonal elements, f_{kj}, are either -1, if releases from the upstream reservoir j are routed into the downstream reservoir k, or 0 otherwise. $\mathbf{L}(t)$ indicates the losses and $\mathbf{I}(t)$ denotes the vector of uncontrollable inflows to the system at the period t. The inflows are assumed to be replaced by forecasted values which can be calculated using techniques described by Hipel and McLeod (1994). This allows a deterministic approach to be undertaken.

The storage and release variables are subject to lower and upper bounds, due to physical constraints of the system:

$$\mathbf{l_S} \leq \mathbf{S(t)} \leq \mathbf{u_S}, \quad (2)$$

$$\mathbf{l_R} \leq \mathbf{R(t)} \leq \mathbf{u_R}. \quad (3)$$

Without loss of generality, the variables can be adjusted to set the lower bounds equal to zero.

For many reservoir systems, the maximization of hydropower generation appears to be the primary objective. The hydroelectric generating function, $\mathbf{P}(t)$, is a nonlinear function of the release and the water head, $\mathbf{H}(t)$, which itself is a function of the storage. So, the first objective can be stated as:

$$\max \quad \mathbf{J_1} = \sum_{i=1}^{K} \sum_{t=1}^{T} \mathbf{P}_i(t), \quad \text{where} \quad (4)$$

$$\mathbf{P_i(t)} = \gamma\ \mathbf{H_i(t)}\ \mathbf{R_i(t)}\ \mathbf{e_i}\ , \quad (5)$$

where γ indicates the specific weight of water and $\mathbf{e_i}$ is the efficiency of the i-th power generating system. The following objectives could also be considered in an optimization model:

1. Satisfying water supply demand at the point j $(\mathbf{J}_2)$:

$$\sum_{\mathbf{i \in h(j)}} (\mathbf{R_i(t)} + \mathbf{D_j(t)}) \geq \sum_{\mathbf{k \in g(j)}} \mathbf{W_k(t)}, \quad (6)$$

where $\mathbf{h}(j)$ is the set of reservoirs upstream of point j. $\mathbf{W}_k(t)$ denotes the nonpower water demand and $\mathbf{g}(j)$ is the set of demand points down to and including point j.

2. Maximization of the firm power generated at each time period $(\mathbf{J}_3)$:

$$\max \quad \{\min_{1 \leq t \leq T} \quad \sum_{i=1}^{K} \mathbf{P}_i(t)\}. \quad (7)$$

3. Maximizing the reliability for acheiving total target release, $\mathbf{TR}(t)$, at the downstream area (if the inflows are assumed to be random variables):

$$\max\ \mathbf{J}_4 = \Pr\{\sum_{t=1}^{T} \mathbf{TR}(t) - \sum_{i=1}^{K} (\mathbf{S}_i(1) - \mathbf{S}_i(T+1)) + \sum_{i=1}^{K} \sum_{t=1}^{T} \mathbf{D}_i(t) \leq \sum_{i=1}^{K} \sum_{t=1}^{T} \mathbf{I}_i(t)\} \geq (1-p),$$

where p denotes the risk level of violating this target.

4. Maximization of a firm flow for low flow augmentation at the downstream area j $(\mathbf{J}_5)$:

$$\max \ \{\min_{1\le t\le T} \ \mathbf{R}_j(t)\}. \tag{8}$$

5. Minimization of evaporation losses:

$$\min \ \mathbf{J}_6 = \sum_{i=1}^{K}\sum_{t=1}^{T}\mathbf{L}_i(t), \ \text{where} \tag{9}$$

$$\mathbf{L}_i(t) = \mathbf{er}(t)\ \mathbf{AR}_i(t), \tag{10}$$

where $\mathbf{er}(t)$ is the monthly average evaporation rate and $\mathbf{AR}_i(t)$ represents the surface area of reservoir i. The relationship between the area and the storage can be found using regression analysis.

The ϵ-constraint formulation of the MROP problem is then given by:

$$(\text{MROP}) \quad \max \quad \mathbf{J}_1 \tag{11}$$

$$\text{s.t. :} \quad \mathbf{F}_i \ge \epsilon_i; \ \ i = 2,3,\cdots,6 \tag{12}$$

and subject to (1) to (3). The feasible ranges of target levels ϵ_i's are obtained by solving the problem with the individual corresponding objective functions. A nondominated solution set (Pareto inferior) is generated by changing ϵ_i's in their feasible ranges.

This model contains the main features of a complicated reservoir system. It can be successively linearized by taking the first-order Taylor series approximation of nonlinear functions. Interior-point methods (IPM's) are capable of solving the resulting general LP problem efficiently. However, a simplified LP version of this model is used in the following sections. All the constraints corresponding to the secondary objectives are relaxed. $\mathbf{D}(t)$ is assumed to be contained in $\mathbf{R}(t)$ and $\mathbf{L}(t)$ is neglected.

Linearizing the energy equation (5) results in a linear function of the release as:

$$\mathbf{P_t} = \mathbf{C_t}\ \mathbf{R_t}, \tag{13}$$

in which the constant $\mathbf{C}_t$ would only depend on the estimated storage volumes. Consequently, the MROP model can be reduced to a sequence of LP problems with bounded variables as:

(LMROP)

$$\max \ J = \sum_{k=1}^{K}\sum_{t=1}^{T}\mathbf{C}_k(t)\ \mathbf{R}_k(t) \tag{14}$$

s.t. :

$$\mathbf{S}_k(t) = \mathbf{S}_k(t-1) + \mathbf{I}_k(t) - \mathbf{R}_k(t) + \sum_{\substack{j=1\\ j\ne k}}^{K} f_{kj}\mathbf{R}_j(t) \tag{15}$$

$$0 \le \mathbf{S}_k(t) \le \mathbf{u}_S^k \tag{16}$$

$$0 \le \mathbf{R}_k(t) \le \mathbf{u}_R^k \tag{17}$$

$$t = 1,\cdots,T; \quad k = 1,\cdots,K\ .$$

3 SOLUTION METHODOLOGY

The field of interior-point methods was activated by Karmarkar's projective algorithm (Karmarkar, 1984). Since then, there have been many advances to the theory and implementation of IPM. Primal-dual path-following and potential reduction methods are two classes of IPM which both have proven to possess low complexity and perform very well in practice. Kojima et al. (1989) and Monteiro and Adler (1989) developed the first polynomial-time primal-dual interior-point algorithm. The algorithm has been revised by other researchers including Kojima et al. (1991) and Mizuno (1992).

Consider the standard primal LP problem with bounded variables $\mathbf{x}$:

$$\min \quad \mathbf{c^T x} \tag{18}$$

$$\text{s.t. :} \quad \mathbf{Ax} = \mathbf{b} \tag{19}$$

$$\mathbf{x} + \mathbf{z} = \mathbf{u} \tag{20}$$

$$\mathbf{x} \ge \mathbf{0}, \ \mathbf{z} \ge \mathbf{0}, \tag{21}$$

where $\mathbf{A} \in \mathbf{R^{m\times n}}, \mathbf{b} \in \mathbf{R^m}$, and $\mathbf{c} \in \mathbf{R^n}$. The matrix $\mathbf{A}$ is assumed to have full row rank. $\mathbf{u}$ is the vector of upper bounds and $\mathbf{z}$ denotes the associated slack variables. The corresponding dual problem is given by:

$$\max \quad \mathbf{b^T y} - \mathbf{u^T w} \tag{22}$$

$$\text{s.t. :} \quad \mathbf{A^T y} - \mathbf{w} + \mathbf{s} = \mathbf{c} \tag{23}$$

$$\mathbf{w} \ge \mathbf{0}, \ \mathbf{s} \ge \mathbf{0}, \tag{24}$$

where $\mathbf{w}$ and $\mathbf{y}$ are dual variables and $\mathbf{s}$ represents dual slack variables. It is also assumed that there exists an interior solution in the primal-dual feasible region.

In the original primal-dual algorithms, it was assumed that the given initial solution $(\mathbf{x^0}, \mathbf{s^0})$ is both primal and dual feasible. Then, researchers developed infeasible-interior-point (IIP) algorithms to start directly from an infeasible point. The algorithms have been studied by several researchers including Lustig (1988) and Lustig et al. (1992) and have turned out to be highly efficient in practice. Zhang (1992) and later Mizuno (1992) have proved the polynomial-time convergence of certain IIP algorithms.

The idea of using a potential function as a measure of progress of an LP algorithm was first introduced by Karmarkar (1984). Potential reduction methods are motivated by searching along the projected gradient of an appropriate potential function to directly reduce that function at each iteration. In this section, the derivation of the search direction for an LP problem with upper bounds is explained first. Then, the method of choosing the stepsizes is discussed.

Applying Newton's method to the perturbed system of first-order optimality conditions for the primal and dual problems results in:

$$\mathbf{dx} = -\mathbf{P}_{\bar{\mathbf{A}}}\Theta(\rho(\mu) + \mathbf{r_d}) + \bar{\mathbf{A}}^{\mathbf{T}}(\bar{\mathbf{A}}\bar{\mathbf{A}}^{\mathbf{T}})^{-1}\Theta^{1/2}\mathbf{r_p}, \quad (25)$$

where $\mathbf{dx}$ is the search direction along $\mathbf{x}$ and $\rho(\mu) = \gamma\mu(\mathbf{Z^{-1}} - \mathbf{X^{-1}})\mathbf{e} - (\mathbf{W} - \mathbf{S})\mathbf{e}$. The parameter μ is the average duality gap, defined by $\mu = (\mathbf{x^T s} + \mathbf{z^T w})/2n$, γ is the centering parameter and $\mathbf{e}$ is a vector of ones with appropriate size. $\bar{\mathbf{A}}$ is the scaled matrix of $\mathbf{A}$, $\bar{\mathbf{A}} = \mathbf{A}\Theta^{1/2}$, in which Θ is defined as $\Theta = (\mathbf{X^{-1}S} + \mathbf{Z^{-1}W})^{-1}$. $\mathbf{P}_{\bar{\mathbf{A}}}$ is the projection matrix onto the null space of $\bar{\mathbf{A}}$, given by $\mathbf{P}_{\bar{\mathbf{A}}} = \mathbf{I} - \bar{\mathbf{A}}^{\mathbf{T}}(\bar{\mathbf{A}}\bar{\mathbf{A}}^{\mathbf{T}})^{-1}\bar{\mathbf{A}}$. The primal and dual feasibility residuals are respectively expressed as $\mathbf{r_p} = \mathbf{b} - \mathbf{Ax}$, $\mathbf{r_d} = \mathbf{c} - \mathbf{A^T y} + \mathbf{w} - \mathbf{s}$. Once $\mathbf{dx}$ has been computed, the other components of the search direction can be easily obtained by substituting for $\mathbf{dx}$ in the following equations:

$$\mathbf{dz} = -\mathbf{dx}, \quad (26)$$

$$\mathbf{dw} = \gamma\mu\mathbf{Z^{-1}e} - \mathbf{We} + \mathbf{Z^{-1}W dx}, \quad (27)$$

$$\mathbf{ds} = \gamma\mu\mathbf{X^{-1}e} - \mathbf{Se} - \mathbf{X^{-1}S dx}. \quad (28)$$

It is implicitly assumed that the starting point always satisfies $\mathbf{x} + \mathbf{z} = \mathbf{u}$, $\mathbf{z} > \mathbf{0}$.

The method of chosing stepsizes plays a crucial role in distinguishing different IP algorithms.

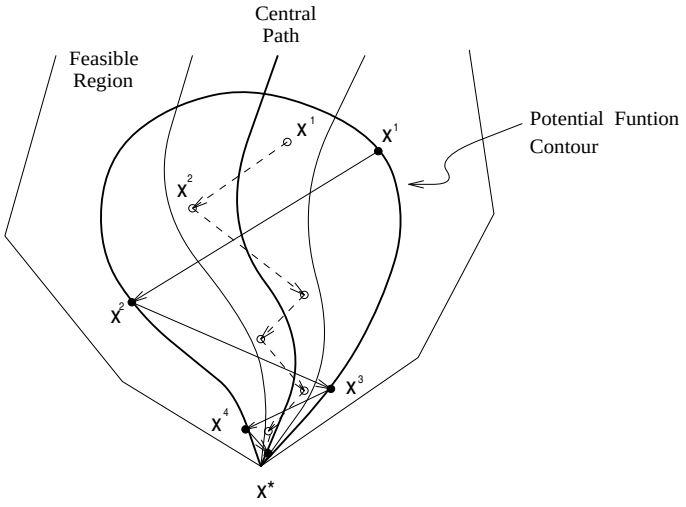

Figure 1: The stepsizes in a constant potential function algorithm.

Tuncel (1992) proposes a constant-potential IP algorithm in which the stepsizes are chosen such that all iterates are kept on the same contour of a potential function. As a result, larger stepsizes can be taken while the global convergence is guaranteed . Figure 1 shows an example of such a contour in a two dimensional space.

Mizuno et al. (1992) suggest a potential function algorithm that may start from an infeasible interior-point. Their algorithm seeks a constant reduction in the following potential function at each iteration:

$$\begin{aligned}\psi(\mathbf{x},\mathbf{y},\mathbf{s}) &= (n+\nu+1)\ln(\mathbf{x^T s} + \mathbf{z^T w}) - n\ln n \\ &\quad - \sum_{j=1}^{n}\ln(\mathbf{x_j s_j}) - \sum_{j=1}^{n}\ln(\mathbf{z_j w_j}) \\ &\quad - \ln(\mathbf{x^T s} + \mathbf{z^T w} - \\ &\qquad \sigma\|(\mathbf{Ax} - \mathbf{b}, \mathbf{A^T y} + \mathbf{s} - \mathbf{c})\|),\end{aligned} \quad (29)$$

where ν and σ are some positive constant. This function involves three major components: the optimality, complementarity and feasibility parts. The balance between the optimality and complementarity parts prevents the generated solutions from getting too close to the boundary before achieving optimality. The inclusion of the infeasibility norm guarantees that the algorithm decreases the infeasibility faster than the duality gap. This prohibits the algorithm from converging to an infeasible complementary solution. In general, any potential function containing the desirable balance among these three parts may be used in the algorithm. The main idea of the proposed method is

to extend the constant-potential algorithm to the infeasible context. Denoting the size of a problem instance by L, the statement of the algorithm is as follows:

Algorithm:

1. Start from any$(\mathbf{x^o}, \mathbf{z^o}, \mathbf{w^o}, \mathbf{s^o})$such that $(\mathbf{x^o})^\mathbf{T}\mathbf{s^o} + (\mathbf{z^o})^\mathbf{T}\mathbf{w^o} \leq \mathbf{2^L}$. Set $k := 0$.

2. While $(\mathbf{x^k})^\mathbf{T}\mathbf{s^k} + (\mathbf{z^k})^\mathbf{T}\mathbf{w^k} > \mathbf{2^{-L}}$, do the following:

3. Set $\gamma_k := \min[0.2, \mu_k]$ and compute $(\mathbf{dx}, \mathbf{dz}, \mathbf{dw}, \mathbf{ds})$ from (25, 26, 27, 28).

4. If $(\mathbf{x^k} + \mathbf{dx}, \mathbf{z^k} + \mathbf{dz}, \mathbf{w^k} + \mathbf{dw}, \mathbf{s^k} + \mathbf{ds})$ is feasible, then it is optimal. Set

$$(\mathbf{x^{k+1}}, \mathbf{z^{k+1}}, \mathbf{w^{k+1}}, \mathbf{s^{k+1}}) := (\mathbf{x^k} + \mathbf{dx}, \mathbf{z^k} + \mathbf{dz}, \mathbf{w^k} + \mathbf{dw}, \mathbf{s^k} + \mathbf{ds})$$

and stop.

5. Otherwise, choose the stepsize $\alpha \in (0, 1)$ such that:

$$\psi(\mathbf{x^k} + \alpha\mathbf{dx}, \mathbf{z^k} + \alpha\mathbf{dz}, \mathbf{w^k} + \alpha\mathbf{dw}, \mathbf{s^k} + \alpha\mathbf{ds}; \nu) = \psi(\mathbf{x^k}, \mathbf{z^k}, \mathbf{w^k}, \mathbf{s^k}; \nu)$$

and find the next iterate by:

$$(\mathbf{x^{k+1}}, \mathbf{z^{k+1}}, \mathbf{w^{k+1}}, \mathbf{s^{k+1}}) := (\mathbf{x^k} + \alpha\mathbf{dx}, \mathbf{z^k} + \alpha\mathbf{dz}, \mathbf{w^k} + \alpha\mathbf{dw}, \mathbf{s^k} + \alpha\mathbf{ds}).$$

6. Set $k := k + 1$.

7. End

This algorithm enjoys many attractive features. First, it may start from any infeasible initial solution and thus avoids the computational difficulties associated with using artificial variables. Secondly, it benefits from the nice properties of the potential function contour. Therefore, larger stepsizes can be taken relative to the path-following algorithms. In practice, these properties would usually coincide with faster convergence of the algorithm.

Monteiro and Wright (1993) present a superlinearly convergent IIP algorithm that can be used for solving LP problems. The authors show that Tuncel's constant-potential algorithm (1992) takes equal or larger stepsizes than those taken by their algorithm and thus achieves the superlinear convergence. Their results certainly support the capability of the proposed algorithm in solving practical problems with a superlinear rate of convergence.

4 EXPLOITING PROBLEM STRUCTURE

Every implementation of an interior-point algorithm requires a linear system of equations (so-called as normal equation) to be solved at each iteration. Solving this linear system efficiently, influences the overall performance of the algorithm. The staircase structure of a multi-stage operation problem introduces a very sparse linear system at each iteration of the algorithm. A thorough examination of the sparse matrix arising from this application reveals how one can take advantage of the problem structure to solve the normal equation effectively. Rewrite the LMROP problem in a more compatible form with the standard LP notation as:

$$\max \quad \mathbf{c}_R^T \mathbf{x}_R \tag{30}$$

$$\text{s.t. :} \quad \mathbf{A}_S\mathbf{x}_S + \mathbf{A}_R\mathbf{x}_R = \mathbf{b}, \tag{31}$$

$$0 \leq \mathbf{x}_S \leq \mathbf{u}_S, \tag{32}$$

$$0 \leq \mathbf{x}_R \leq \mathbf{u}_R, \tag{33}$$

where $\mathbf{A}_S$ and $\mathbf{A}_R$ are the coefficient matrices associated with storage and release variables, respectively. There are two sets of variables in the model corresponding to storage volumes, $\mathbf{x}_S$, and release rates, $\mathbf{x}_R$. The vector $\mathbf{b}$ indicates the inflows and $\mathbf{u}_S$ and $\mathbf{u}_R$ are as defined earlier in Section 2.

The matrix $\mathbf{A}_S$ is a $(K \times K)$ block diagonal matrix with each block itself being a $(T \times T)$ bidiagonal matrix, as shown below. The structure of $\mathbf{A}_R$ depends on the system configuration matrix, $\mathbf{F} = [f_{ij}]$, defined in Section 2. In general, $\mathbf{A}_R$ is a $(K \times K)$ block lower triangular matrix, as follows:

$$\overbrace{\begin{pmatrix} 1 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{pmatrix}}^{\mathbf{A_S}} \quad \overbrace{\begin{pmatrix} B_{11} & & & \\ B_{21} & B_{22} & & \\ \vdots & \vdots & \ddots & \\ B_{K1} & B_{K2} & \cdots & B_{KK} \end{pmatrix}}^{\mathbf{A_R}}$$

where $\mathbf{B}_{ij} = f_{ij}\,\mathbf{I}$, and $\mathbf{I}$ denotes a $(T \times T)$ identity matrix.

As discussed earlier, the computation of the search direction at each iteration of an IP algorithm requires solving the following normal equation:

$$(\mathbf{A}\boldsymbol{\Theta}\mathbf{A}^T)\,\mathbf{d} = \xi, \tag{34}$$

where $\boldsymbol{\Theta}$ is a diagonal matrix, $\mathbf{d}$ is the search direction and ξ is a constant right hand side. The structure of matrix $\mathbf{A}$ allows one to partition the coefficient matrix $(\mathbf{A}\boldsymbol{\Theta}\mathbf{A}^T)$ as:

$$(\mathbf{A}_S\boldsymbol{\Theta}^S\mathbf{A}_S^T + \mathbf{A}_R\boldsymbol{\Theta}^R\mathbf{A}_R^T)\,\mathbf{d} = \xi. \tag{35}$$

The matrix $(\mathbf{A}_S\boldsymbol{\Theta}^S\mathbf{A}_S^T)$ is a block diagonal matrix with its components being as:

$$\mathbf{D}_i\boldsymbol{\Theta}_i^S\mathbf{D}_i^T = \begin{pmatrix} \theta_1^S & -\theta_1^S & & & \\ -\theta_1^S & \theta_1^S+\theta_2^S & -\theta_2^S & & \\ & \ddots & \ddots & \ddots & \\ & -\theta_{T-2}^S & \theta_{T-2}^S+\theta_{T-1}^S & -\theta_{T-1}^S & \\ & & -\theta_{T-1}^S & \theta_{T-1}^S+\theta_T^S \end{pmatrix}$$

It is easy to show that $(\mathbf{A}_R\boldsymbol{\Theta}^R\mathbf{A}_R^T)$ is a symmetric block matrix whose diagonal component of row k equals $\boldsymbol{\Theta}_i^R + \sum_{j=1}^{i-1} f_{ij}\boldsymbol{\Theta}_j^R$. The off-diagonal block (i,j) of this matrix is $-f_{ij}\boldsymbol{\Theta}_i^R$.

Given the configuration of the reservoir system, a special data structure for solving the normal equation (35) can be designed. This contains a great potential of speeding up the algorithm for solving very large LMROP problems.

5 A GOOD INITIAL SOLUTION

Although the proposed algorithm can handle any arbitrary initial solution, it would be more advantageous to find a good starting point. One would like to find an initial set of primal-dual solutions which would be as close to the feasible region as possible, be near the central path and have a good objective value. It turns out that there is always a good starting solution for LMROP problems. The following solution satisfies the upper bound constraints in the LMROP model. However, it might violate the equality constraints due to having very high inflows. In that case, one could let the algorithm handle the initial infeasibility and find a feasible solution.

$$\mathbf{S}_k(t) = \mathbf{S}_k^0 \tag{36}$$

$$\mathbf{R}_k(t) = \max\{\mathbf{u}_R^k,\ \mathbf{I}_k(t) + \sum_{j=1}^{K} f_{kj}\mathbf{R}_j(t)\} \tag{37}$$

Denoting the dual variables corresponding to primal equality constraints and upper bound constraints by $\mathbf{y}$, $\mathbf{w}_S$ and $\mathbf{w}_R$, respectively, the dual LMROP problem may be written as:

$$\min \quad \mathbf{b}^T\mathbf{y} + \mathbf{u}_S^T\mathbf{w}_S + \mathbf{u}_R^T\mathbf{w}_R \tag{38}$$

$$\text{s.t. :} \quad \mathbf{A}_S^T\mathbf{y} + \mathbf{w}_S - \mathbf{z}_S = 0 \tag{39}$$

$$\mathbf{A}_R^T\mathbf{y} + \mathbf{w}_R - \mathbf{z}_R = \mathbf{c}_R \tag{40}$$

$$\mathbf{w}_S, \mathbf{w}_R, \mathbf{z}_S, \mathbf{z}_R \geq 0 \tag{41}$$

where $\mathbf{z}_S, \mathbf{z}_R$ are the dual surplus variables. For the dual problem, the following solution is always feasible:

$$\mathbf{z}_S, \mathbf{z}_R = \text{arbitrary positive numbers} \tag{42}$$

$$\mathbf{y}_1 = \mathbf{y}_2 = \cdots = \mathbf{y}_m = \epsilon \tag{43}$$

$$\mathbf{w}_S = \mathbf{z}_S \tag{44}$$

$$\mathbf{w}_R = \max\{\epsilon,\ \mathbf{c}_R + \mathbf{z}_R\} \tag{45}$$

where ϵ is a small positive number. Since $\mathbf{z}_S$ and $\mathbf{z}_R$ may be chosen arbitrarily, one can choose them so that the above solution lies partially on the central path, i.e., $\mathbf{z}_S = \mathbf{x}_S^{-1}$, $\mathbf{z}_R = \mathbf{x}_R^{-1}$, where $\mathbf{x}_S$ and $\mathbf{x}_R$ are the initial solutions to the primal problem. In order to make this solution as close as possible to the optimum, the constant ϵ is chosen to be a very small positive number. As a result, the set of primal-dual initial solution introduced here, is always dual feasible and primal feasible if a flooding inflow does not occur. It is also close to the central path, thus allowing one to take large stepsizes and attain the optimal solution faster.

6 NUMERICAL RESULTS

In order to test the behavior of the proposed algorithm on a typical LMROP formulation, a benchmark problem was adopted from Chara and Pant (1984). Figure 2 shows the physical configuration of the reservoir system. In these experiments, the effects of exploiting the LMROP structure and starting from a good initial solution, were investigated. In order to test large size problems, the

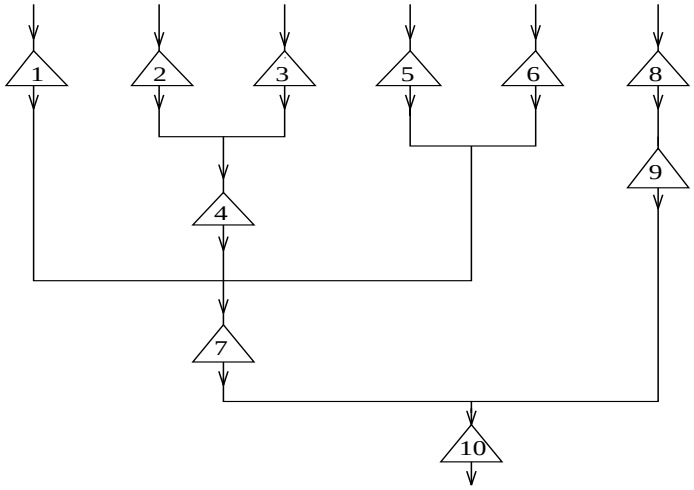

Figure 2: Network of the reservoir system.

Table 1: Computational results on solving the problem using UBPD and ESPD.

T	UBPD		ESPD	
	# of Itns	# of Flops	# of Itns	# of Flops
12	18	2.456E+07	18	9.774E+05
24	17	8.662E+07	17	2.109E+06
48	21	6.477E+08	21	5.915E+06

benchmark problem was simulated for longer time periods. The simulated problems had $10 \times T$ constraints and $2 \times 10 \times T$ variables, where T denotes the number of periods (in months). The largest problem solved was of size 12000×24000. A constant-potential IIP algorithm was implemented for handling the upper bounds in an effective way. In all experimental runs, the primal and dual feasibility as well as the optimality tolerances were set equal to 10^{-6}.

The first code (named UBPD) uses Matlab decomposition routine to solve the normal equation. It treats the coefficient matrix **A** as a general dense matrix. This code is modified to take advantage of the special structure of the matrix **A** associated with LMROP problems as explained in Section 4. The modified code (called ESPD) is expected to solve the normal equation much faster than UBPD at each iteration, thus solving a typical LMROP problem in much less overall time.

Table 1 compares the number of flops (floating point operations) required to solve the problem with UBPD and ESPD. The same initial solution was used when running the ESPD code. As expected, the number of iterations taken by both codes are the same. However, the number of flops that ESPD requires to solve these problems ranges from 1.0% to 3.9% of those needed by UBPD. The second set of experiments investigates the computational advantages of starting the algorithm from a good initial solution, as explained in Section 5. Table 2 shows that a remarkable saving in the number of iterations and flops is attained. The most interesting result of these experiments is that the number of iterations required by the algorithm grows very slowly as the problem size becomes larger. This is a good indication of the capability of the algorithm for solving large practical problems.

Table 2: Computational results on solving the problem using ESPD from different initial solution.

T	Random Initial Solution		Good Initial Solution	
	#of Itns	#of Flops	#of Itns	#of Flops
12	18	9.923E+05	18	9.774E+05
24	22	2.778E+06	17	2.109E+06
48	24	6.898E+06	21	5.915E+06
120	32	2.565E+07	28	2.197E+07
240	40	5.002E+07	30	3.645E+07
480	44	1.118E+08	32	7.899E+07
1200	45	2.886E+08	32	1.993E+08

7 CONCLUSION

This research introduces an application of IPM to the field of reservoir operation planning. A constant-potential interior-point algorithm is proposed to overcome the dimensionality problem. The preliminary numerical results demonstrate that a large amount of computational saving can be achieved by exploiting the problem structure and by starting from a good initial solution. A future avenue of research would be to consider the stochasticity of the inflows into the MROP model. Then, the resulting structure can be exploited to efficiently solve the normal equation.

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