Evaluating triple integrals.

A) Rectangular coordinates: $\iiint_R f(x,y,z)\,dx\,dy\,dz$.

METHOD 1:

* To find the bounds for x & y: project R onto the xy-plane to get a region D which bounds the possible values of x & y.

* To find the bounds of z: take a vertical cross-section

NOTE: the bounds of z are ONLY determined by surfaces whose equations involve z.

METHOD 2:

* Find all possible values of z in R: $\boxed{a \leq z \leq b}$.
* For a fixed $z_0 \in [a,b]$, take a horizontal slice

$$S = \{z = z_0\} \cap R$$

of the region, and describe S.

B) Cylindrical coordinates: $\iiint_{R_{r\theta z}} f(r\cos\theta, r\sin\theta, z)\cdot \overset{\text{Jac}}{\overset{\downarrow}{r}}\,dz\,dr\,d\theta$.

METHOD 1:

* To find the bounds for r & θ: project R onto the xy-plane, and describe the region D you obtain in polar coordinates.

* To find the bounds of z: take a vertical cross-section.

NOTE: • the bounds of z are ONLY determined by surfaces whose equations involve z

• write these equations in cylindrical coordinates.

METHOD 2:

* Find all possible values of z in R: $\boxed{a \leq z \leq b}$.
* For fixed $z_0 \in [a,b]$, take a horizontal slice

$$S = \{z = z_0\} \cap R$$

of the region, and describe S in polar coordinates.

C) Spherical coordinates:

$$\iiint_{R_{\rho\phi\theta}} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi) \cdot \underbrace{\rho^2\sin\phi}_{\text{Jac.}} \, d\rho\, d\phi\, d\theta$$

* Bounds for θ: project R onto xy-plane.
* Bounds for ρ & ϕ: take vertical cross-section.

NOTE: • You will need to convert all the equations into spherical coordinates.

• In spherical coordinates

$$\phi = \arctan\left(\frac{\sqrt{x^2+y^2}}{z}\right).$$