Suggested triple integrals.

1. Express the volume of the following regions as triple integrals in rectangular coordinates.

   (a) R is the region bounded by $z = y$, $z = 2 - y^2$, $z = 0$, $x = 0$, and $x = 2$.

   (b) R is the region in the first octant bounded by $z = x^2$, $z = y^2$, and $z = 1$.

   (c) R is the region bounded by $z = x$, $2x + z = 2$, $y = 0$, $y = 3$, and $z = 0$.

   (d) R is the region bounded by $x + z = 1$, $z = 2y$, $y = x$, and $z = 0$.

2. Compute the volume of the "ice cream cone" bounded by $x^2 + y^2 + z^2 = 2z$ and $z = \sqrt{x^2 + y^2}$ by using (a) cylindrical coordinates, (b) spherical coordinates.

3. Use cylindrical coordinates to solve the following questions.

   (a) Suppose a beehive is shaped like the region R below $z = 4 - x^2 - y^2$ and inside $x^2 + y^2 + z^2 = 4z$, and the number of bees per unit of volume is $f(x, y, x) = 3$ in R. Determine the number of bees in the beehive.

   (b) Compute the volume of the region R inside $x^2 + y^2 + z^2 = 4z$ and below both $z = x^2 + y^2$ and $z = 2$.

4. Use spherical coordinates to solve the following problems.

   (a) Consider a solid that has the shape of the three-dimensional region R lying inside $x^2 + y^2 + z^2 = 2$, outside $x^2 + y^2 + z^2 = 1$, below $z = \sqrt{3x^2 + 3y^2}$, above $z = 0$, and with $x \geq 0$. Suppose that the solid has density $f(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ per unit of volume at every point in R. Find the total mass of the solid.

   (b) Evaluate $\int\int\int_R z dV$, where$R$ is the region inside $x^2 + y^2 + z^2 = 1$ and outside $x^2 + y^2 + z^2 = 2z$.