ERRATUM FOR APPROXIMATING RATIONAL POINTS ON TORIC VARIETIES

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We correct an error in [MS21, Proposition 1.3]: Vojta's Conjecture does not imply canonical boundedness in general. However, the implication does hold for toric varieties. A corrected statement and proof of Proposition 1.3 follows.

Proposition 0.1. *Let X be a smooth projective variety over a number field k, and let $P \in X(k)$ be a k-rational point. If the anticanonical divisor class $-K_X$ is big, then Vojta's Main Conjecture implies that X is canonically bounded at P.*

Proof. Let $\dim X = n$ and fix a place v of k. Let $S = \{v\}$ and let D be the union of any n normal crossings divisors that intersect properly and transversely at P. We claim that there is a constant C such that for all $Q \in X(k)$ outside a proper closed subset Z, we have:

$$m_S(D, Q) \geq -n \log \operatorname{dist}_v(P, Q) + C \tag{0.2}$$

To see this, note that the divisor D has multiplicity at least n at P, by construction. Thus, if $\phi\colon Y \to X$ is the blowup of X at P, the divisor $\phi^*D - nE$ is effective, where E is the exceptional divisor of ϕ. This implies that $m_S(\phi^*D-nE, Q)$ is bounded below independently of Q (see [Vo87, Lemma 1.3.3.(b)]), and so

$$m_S(\phi^*D, Q) \geq m_S(nE, Q) + C$$

for some constant C independent of Q. Equation (0.2) then follows from Lemma 1.3.3.(d) of [Vo87].

Since $-K_X$ is big, we may choose an ample $\mathbb{Q}$-divisor A such that $-K_X - A$ is effective. Fix any $\epsilon > 0$. If Q satisfies Vojta's inequality, then

$$\operatorname{dist}_v(P, Q)^n H_{-K_X}(Q) \geq C_1 H_A(Q)^{-\epsilon} \tag{0.3}$$

for some positive constant C_1, independent of Q. Since $-K_X - A$ is effective, we have

$$H_{-K_X-A}(Q) > C_2 > 0$$

for some positive constant C_2 independent of Q, and thus

$$\operatorname{dist}_v(P, Q)^n H_{-K_X}(Q) \geq C_1 H_A(Q)^{-\epsilon} \geq C_3 H_{-K_X}(Q)^{-\epsilon}$$

from which we deduce

$$\operatorname{dist}_v(P, Q)^n H_{-K_X}(Q)^{1+\epsilon} \geq C_3 > 0.$$

Therefore

$$\operatorname{dist}_v(P, Q)^{n-\epsilon'} H_{-K_X}(Q) \geq C_4$$

for another positive constant C_4, independent of Q. This implies $\alpha_P(-K_X) \geq n - \epsilon$ for all $\epsilon > 0$. We conclude that $\alpha_P(K_X) \geq n$, as desired. □

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References

[MS21] McKinnon, D. and Satriano, M., *Approximating rational points on toric varieties*, Trans. Amer. Math. Soc. 374 (2021), no. 5, 3557–3577.

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