Intro to Mathematical Notation

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1 Introduction

This document summarizes much of the notation used in Math 137. Developing familiarity with these symbols is like developing vocabulary and grammar in the language of mathematics; these symbols aren't just shorthand for English. They are rigorously defined, communicating something precise. Understanding how they can be used will help to develop an understanding of mathematical logic, objects, and abstractions... not to mention your course notes.

2 Sets

A *set* is a collection of distinct numbers. Sets are written in curly brackets, like

$$\{1, 2, 3\}. \tag{1}$$

We can also name sets and treat them as objects in their own right. Say we let

$$A = \{2, 4, 6\}. \tag{2}$$

Then we call the numbers 2, 4, and 6 *elements* of the set A. Large sets or continuations of sets can be indicated with ellipses. Respectively,

$$\{1, 2, \ldots, n\} \text{ and } \{1, 2, 3 \ldots\}. \tag{3}$$

We can also have sets of numbers which satisfy some property, without having to write each number down, by writing

$$\{x \mid x \text{ has property } P\} \text{ or } \{x : x \text{ has property } P\}. \tag{4}$$

These statements can be read as "the set of numbers x such that x has property P." For example, we can write the set of even numbers as

$$\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\} = \{x \mid x \text{ is even}\}. \tag{5}$$

Symbol	Name	Example	Meaning
$\in$	element of	$x \in A$	x is an element of the set A
$\notin$	not an element of	$x \notin A$	x is not an element of the set A
$\cup$	union	$A \cup B$	$\{\, x \mid x \in A \text{ or } x \in B \,\}$
$\cap$	intersection	$A \cap B$	$\{\, x \mid x \in A \text{ and } x \in B \,\}$
$\setminus$	difference of sets	$A \setminus B$	$\{\, x \mid x \in A \text{ and } x \notin B \,\}$
$\times$	product of sets	$A \times B$	$\{\, (a, b) \mid a \in A,\, b \in B \,\}$
$\subset$	subset	$A \subseteq B$	every element of A is also an element of B
$\supset$	superset	$A \supseteq B$	A contains all the elements of B and more
$\emptyset$	empty set	$\{\ \}$	the set with no elements

Table 1: Set notation. Adapted from [1].

Interval	Corresponding Set
$[a, b]$	$\{\, x \mid a \leq x \leq b \,\}$
(a, b)	$\{\, x \mid a < x < b \,\}$
$[a, \infty)$	$\{\, x \mid x > a \,\}$

Table 2: Interval notation.

2.1 Intervals

Sections of a number line, or intervals, are sets of numbers. We can't enumerate these sets—that is, we can't write them all out explicitly. Instead, we can use the following interval notation.

We call the square brackets *closed* and the round ones *open*. Note that when writing ∞ in an interval, it is always next to an open bracket, since we can't reach infinity in the same way we can reach a finite number. Note also that we can combine the brackets. For example, $[a, b) = \{\, x \mid a \leq x < b \,\}$.

2.2 Special Sets

We regularly reference several important sets.

Symbol	Name	Interpretation
$\mathbb{R}$	real numbers	$\{\, x \mid x \in (-\infty, \infty) \,\}$
$\mathbb{N}$	natural numbers	$\{\, 1, 2, 3, ... \,\} = \{\, x \mid x \text{ is a positive integer} \,\}$
$\mathbb{Z}$	integers	$\{\, x \mid x \text{ is an integer} \,\}$

Table 3: Special sets.

3 Functions

To completely define a function, we need to know its domain (or the space it acts on), its range (or the space it takes arguments to), its name (often f), and its specific form. We can succinctly say that "a function called f takes elements of a set A and transforms them into elements of set B" by writing

$$f : A \to B. \tag{6}$$

We can then write the specific form of the function f as, for example,

$$f(x) = x^2. \tag{7}$$

If a function takes more than one argument, we write that it acts on the product of two spaces (see the above definition of the product of sets). For example, a distance function d on some space D takes two elements of D and returns a positive real number that is the distance between them:

$$d : D \times D \to [0, \infty)]. \tag{8}$$

4 Quantifiers & Implications

We sometimes use special symbols to quantify for which objects a property holds true.

Symbol	Name	Example	Interpretation
$\forall$	for all, for every	$0\,x = 0\;\forall\, x \in \mathbb{R}$	for every real number x, $0\,x = 0$
$\exists$	there exists	$\forall$ car, $\exists$ 4 wheels.	for every car, there exist 4 wheels
$\Rightarrow$	implies	$A \Rightarrow B$	truth of A implies truth of B
$\Leftarrow$	left implication	$A \Leftarrow B$	truth of B implies truth of A
$\Leftrightarrow$	if and only if (iff)	$A \Leftrightarrow B$	truth of A implies B and truth of B implies A

Table 4: Quantifiers and implications.

5 Common Variables and Other Object Names

Many mathematicians have preferences for which variables they use to represent specific objects. For example, one often names function $f(x), g(y), h(z)$ and so on. It is important to note that the *name* of an object does not tell you what *kind* of object it is—that is, not all objects named f are necessarily functions, and not all functions will have a name like f or g. Some functions will be named things like det or log or Θ. That being said, let's look at common variables and related notation.

Functions are often named f, g, and h, with *arguments* like x, y, and z. Other Latin letters like a, b, and c are often used to denote constants. Subscripts are often written with n and m or i and j. We're going to need plenty of variables, so we also use some of the Greek alphabet. Frequently used Greek letters are listed below.

α	alpha	θ	theta	ρ	rho
β	beta	κ	kappa	σ	sigma
γ	gamma	λ	lambda	τ	tau
δ	delta	μ	mu	ϕ	phi
ϵ or ε	epsilon	ν	nu	χ	chi
ζ	zeta	ξ	xi	ψ	psi
η	eta	π	pi	ω	omega

Table 5: Lowercase Greek letters.

Γ	Gamma	Ξ	Xi	Φ	Phi
Δ	Delta	Π	Pi	Ψ	Psi
Λ	Lambda	Σ	Sigma	Ω	Omega

Table 6: Capital Greek letters.

We use boldface or arrows to indicate that an object is a vector:

$$\vec{a} \text{ or } \mathbf{a}. \tag{9}$$

Matrices and sets are usually writen in uppercase, like A and D.

References & Further Reading

[1] C. Batty, N. Woodhouse, *et al. How do undergraduates do mathematics? A guide to studying mathematics at Oxford University.* https://www.maths.ox.ac.uk/system/files/attachments/study_public_1.pdf. 2014.

[2] *List of LaTeX Mathematical Symbols.* OEIS. https://oeis.org/wiki/List_of_LaTeX_mathematical_symbols. Accessed September 2017.

[3] Antonella Cupillari. *The Nuts and Bolts of Proofs.* Elsevier Academic Press (2005). UWaterloo library call number QA9.54 .C86 2005.