The time-delayed inverted pendulum: Implications for human balance control

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Abstract

The inverted pendulum is frequently used as a starting point for discussions of how human balance is maintained during standing and locomotion. Here we examine three experimental paradigms of time–delayed balance control: 1) the mechanical inverted time–delayed pendulum, 2) stick balancing at the fingertip, and 3) human postural sway during quiet standing. Measurements of the transfer function (mechanical stick balancing) and the two–point correlation function (Hurst exponent) for the movements of the fingertip (real stick balancing) and the fluctuations in the center of pressure (postural sway) demonstrate that the upright fixed-point is unstable in all three paradigms. These observations imply that the balanced state represents a more complex and bounded time-dependent state than a fixed–point attractor. Although mathematical models indicate that a sufficient condition for instability is that the time delay to make a corrective movement, τ_n, be greater than a critical delay, τ_c, that is proportional to the length of the pendulum, this condition is satisfied only in the case of human stick balancing at the fingertip. Thus it is suggested that a common cause of instability in all three paradigms stems from the difficulty controlling both the angle of the inverted pendulum and the position of the controller simultaneously using time-delayed feedback. Considerations of the problematic nature of control in the presence of delay and random perturbations ("noise") suggests that neural control for the upright position likely resembles an adaptive–type controller in which the displacement angle is allowed to drift for small displacements with active corrections made only when θ exceeds a threshold. This mechanism draws attention to an overlooked type of passive control that arises from the interplay between retarded variables and noise.

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A high proportion of falls in the elderly occur while walking [1]. Although some of these falls can be attributed to "slips and trips", for many the immediate cause is unknown. A first step towards the development of strategies to minimize the risk of falling in the elderly is to understand how balance is maintained during locomotion. The question of how best to stabilize the upright position of an inverted pendulum, an unstable fixed–point, is a classic problem in control theory [2] with applications ranging from the Segway [3] to missile guidance systems [4] to lifting cranes [5]. Typically overlooked in biomechanical applications of the inverted pendulum to human balance control are the effects of time-delays [6–11]. These delays arise because there is a significant time interval between when a variable is measured and when corrective forces are applied. Here we review issues that arise in determining the stability of the time-delayed inverted pendulum and compare the observations to three paradigms of balance control: 1) the mechanical inverted time–delayed pendulum [12–16], 2) stick balancing at the fingertip [17–25], and 3) postural sway during quiet standing [26–32]. It is argued that misconceptions about balance control arise when the effects of time delay are ignored [33–35]. We draw attention to a novel "passive control" mechanism for maintaining balance that arises from the interplay between random perturbations ("noise") and delay [35–38]. Thus it is possible that interactions between the sole of the foot and the walking surface can, on the one hand, be the cause of the fall and, on the other, be a stabilizing mechanism for minimizing the risk of falling.

I. INTRODUCTION

Concepts derived from considerations of the inverted pendulum arise frequently in discussions of the control of human balance [30, 31, 39] and walking [40–43]. This approach has been particularly successful in understanding the changes in the kinetic and potential energy that occur during human locomotion [44, 45]. However applications to the study of human gait and balance stability are made difficult because the precise identity of the controller is not known, and hence the full dynamical system can not be written down. Consequently the approach has been to use experimental observations to try to determine the nature of the control strategies. Typically these findings are interpreted in the context of models having the general form of an inverted pendulum, such as

$$\ddot{\theta}(t) + \beta\dot{\theta}(t) - \alpha\theta(t) = F_{\text{control}}(t) \tag{1}$$

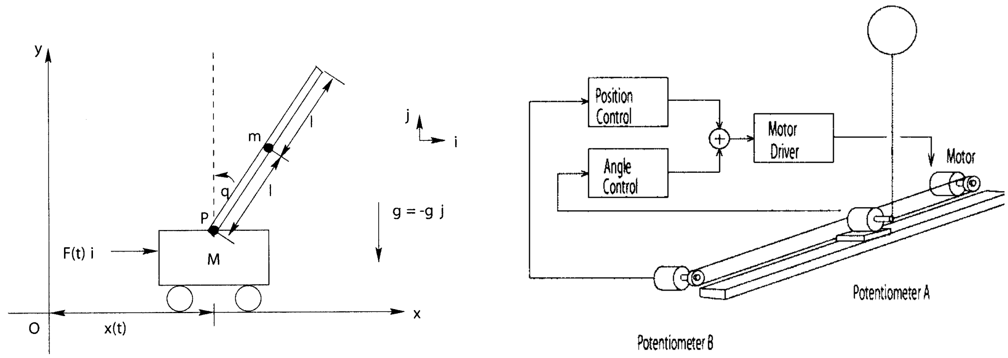
where α, β are positive constants chosen so that in the absence of control the fixed point is unstable, θ is the vertical displacement angle ($\theta = 0$ corresponds to the upright position, hence the "$-$"), and F_{control} describes the proposed feedback controller. Particular attention has been given to the fact that neural feedback control mechanisms are time-delayed (neural latencies are $\sim 100 - 500$ms) [6–8, 10, 11, 46]. Consequently (1) becomes

$$\ddot{\theta}(t) + \beta\dot{\theta}(t) - \alpha\theta(t) = F_{\text{control}}(t - \tau) \tag{2}$$

where τ is the time delay. Moreover, it is increasingly being recognized that uncontrolled perturbations ("noise"), likely related to muscle activity [47, 48], can play important roles in maintaining balance [35–38]. Although it is permissible to ignore the effects of time delays when considering issues related to the energetics of locomotion, considerations of the effects of time delays and noise are essential for understanding the stability of balance and gait [6].

To date there have been no attempts to directly compare the dynamics of mechanical pendulums stabilized by delayed feedback [13, 14, 16] to those observed for well studied human paradigms of balance control, namely, stick balancing at the fingertip [17–25] and postural control during quiet standing [26–32]. Such comparisons are essential in order to identify those aspects of the control that are in common, and hence are understood, from those aspects of control that are different and hence require further attention. Here we explore whether the balanced state represents a fixed–point attractor or a more complex and bounded time–dependent state.

We organize our discussion as follows. In Section II we briefly review the feedback stabilization of a pendulum attached to a cart at a pivot point and then, in Section III, include a time delay in the feedback. An important concept in these mathematical studies is the relative magnitude of the feedback delay, τ_n, versus a critical delay, τ_c, which is proportional to one–half the length of the pendulum. Although $\tau_n > \tau_c$ is sufficient to guarantee instability, $\tau_n < \tau_c$ does not necessarily guarantee stability. In Sections IV and V we examine three paradigms of balance control: the mechanical inverted pendulum with time–delayed feedback (Sections IVA and VA), stick balancing at the fingertip (Sections IVB and VB),



(a) (b)

FIG. 1: a) Schematic representation of inverted pendulum stabilized by the movements of a cart. M is the mass of the cart and P is the pivot point of the pendulum. See text for definition of other parameters. b) Implementation of delayed feedback control of an inverted pendulum that utilizes the carriage mechanism of a DC–motor–operated plotter [14] (see METHODS for details).

and postural sway during quiet standing (Sections IVC and VC). In each case we conclude that the upright fixed–point is unstable; however, only in the case of human stick balancing is $\tau_n \geq \tau_c$. These observations strongly support previous suggestions that the balanced position does not simply represent a noisy fixed–point attractor, but rather a more complex and bounded behavior [20, 26–30, 32, 49–54]. Finally in Section VI we argue that the differences between human and mechanical control of balance because the presence of time delays and random perturbations ("noise") place severe restrictions on the nature of feasible control strategies. In this way we draw attention to a number of fundamental problems for balance control with time–delayed feedback that, up until this time, have been overlooked by the neuroscience and bio–mechanics communities.

II. INSTANTANEOUS CONTROL ($\tau = 0$)

The standard engineering approach to the problem of stabilizing an inverted pendulum is depicted schematically in Figure 1(a). The pendulum is attached to a cart by means of a pivot, which allows the pendulum to rotate freely in the xy plane. Neglecting friction in the pivot, the equations of motion for the full system are:

$$\begin{aligned} (m+M)\ddot{x} + F_{\text{fric}} + m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}^2\sin\theta &= F_{\text{control}} \\ m\ell\ddot{x}\cos\theta + \tfrac{4}{3}m\ell^2\ddot{\theta} - mg\ell\sin\theta &= 0. \end{aligned} \tag{3}$$

where M is the mass of the cart, ℓ is half the length of the pendulum, i.e., the distance from the pivot to the center of mass of the pendulum, and F_{control} represents the force that is applied to the cart in the x direction for the purpose of keeping the pendulum upright. The term F_{fric} represents friction between the cart and the track and can be quite complicated for some experimental setups [13, 16]. In the following we will take $F_{\text{fric}} = \delta\dot{x}$, i.e. simple viscous friction, for concreteness.

When F_{control} is chosen based on the current values of the system variables, it can be shown that one can always find a linear feedback law which depends on all four degrees of freedom that will stabilize the pendulum in the inverted position [2]. This can be seen as follows. Let

$$F_{\text{control}} = k_1 x + k_2\theta + k_3\dot{x} + k_4\dot{\theta}, \tag{4}$$

where the k_j are to be determined. Then the characteristic equation of the linearization of the equations of motion (3) about the equilibrium point corresponding to the upright position of the pendulum is

$$\begin{aligned}\Delta(\lambda) \;=\; & \ell(m+4M)\lambda^4 + (3k_4 - 4\ell k_3 + 4\ell\delta)\lambda^3 + (3k_2 - 4\ell k_1 - 3(m+M)g)\lambda^2 \\ & +3(k_3-\delta)g\lambda + 3k_1 g.\end{aligned} \tag{5}$$

The Routh-Hurwitz criterion states that a necessary condition for all the roots of the above polynomial to be in the left-half-plane is that all the coefficients of λ be non-zero and have the same sign [2]. The coefficient of the fourth-order term of characteristic equation (5) is positive. Therefore stability of the upright position requires that the coefficients of all the lower terms also be positive. This observation leads to the following constraints on the state-feedback gain parameters [7, 10]:

$$k_1 > 0, \qquad k_3 > \delta, \tag{6}$$

and k_2 and k_4 are bounded by k_1 and k_3:

$$k_2 > \frac{4\ell}{3}k_1 + (m+M)g, \qquad k_4 > \frac{4\ell}{3}(k_3-\delta). \tag{7}$$

A variety of methods have been developed to determine the "optimal" choices of the k_j which satisfy these criteria (see, e.g. [2]). Note that for this model, when the feedback control stabilizes the pendulum in the upright position ($\theta = 0$) the position of the cart is fixed at $x = 0$. It is not possible to stabilize the pendulum at $\theta = 0$ with the cart in an arbitrary position. In the terminology of control theory, the system (3) with feedback control (4) is *stabilizable* but not *controllable*.

Two approaches can be taken to simplify the analysis for stabilization of the upright position of the inverted pendulum. First, we can neglect the dynamics of the cart. This corresponds to taking $k_1 = 0, k_3 = \delta$ in the feedback law and assuming the mass of the cart is much less than that of the pendulum, $M + m \approx m$, and produces the model [7]

$$(4 - 3\cos^2\theta)\,\ddot{\theta} + \frac{3}{2}\sin 2\theta\,\dot{\theta}^2 - \frac{3g}{\ell}\sin\theta = -\frac{3}{m\ell}\cos\theta F_{\text{control}} \tag{8}$$

with feedback force:

$$F_{\text{control}} = k_2\theta + k_4\dot{\theta}.$$

The constraints (7) for stabilizing the pendulum in the inverted position become

$$k_2 > mg, \qquad k_4 > 0. \tag{9}$$

which agree with those derived in [11]. For the discussion that follows (see RESULTS) we note the equation for the cart becomes

$$\ddot{x} = g\tan\theta - \frac{4}{3}\ell\sec\theta\,\ddot{\theta}.$$

Thus when the pendulum is at the inverted position, $\theta = 0, \dot{\theta} = \ddot{\theta} = 0$, the cart is not at a fixed position but moves with some constant speed.

An alternate approach is to assume that the inverted pendulum is stabilized not by the application of forces at the base, but by the direct application of torque at the pivot. In this case the model is very simple

$$\frac{4}{3}m\ell^2\ddot{\theta} - mg\ell\sin(\theta) = T_{\text{control}} \tag{10}$$

where the linear feedback control torque is

$$T_{\text{control}} = q_2\theta + q_4\dot{\theta}.$$

The linearization of equation (10) about $\theta = 0$, is very similar to that of equation (8). Thus the analysis of [6, 11] may be easily restated for this equation. In particular, the pendulum will be stabilized in the upright position for any choice of feedback satisfying

$$q_2 < -mg\ell, \qquad q_4 < 0.$$

It is important to note that in all of these approaches the criteria are derived using linearization and hence the control is applied locally. Thus for stabilization of the inverted position to be possible it is necessary to first bring the pendulum close to the upright position (θ is small). If a perturbation pushes the pendulum sufficiently far from the upright position the feedback control will fail. This is also true when the feedback is time-delayed.

III. STABILIZATION WITH DELAYED FEEDBACK

From the point of view of the human body, the only way to implement the feedback control, $F_{\rm control}$, instantaneously is to assume that it is due to the biomechanical properties of the joints, connective tissues, etc. Indeed, historically it was thought that balance control could be entirely due to these biomechanical properties [30, 31, 55]. However, subsequent measurements demonstrated that these forces alone were not sufficient to effectively maintain balance [56, 57]. Neural feedback control mechanisms for balance are time-delayed. In other words there is a significant time interval between when the variables are *measured* and when the forces are applied. Consequently the force applied to the cart becomes

$$F_{\rm control} = k_1 x(t-\tau) + k_2 \theta(t-\tau) + k_3 \dot{x}(t-\tau) + k_4 \dot{\theta}(t-\tau), \tag{11}$$

where it is assumed that the measurements all occur at the same time. The approaches taken to choose the k_j to stabilize the pendulum depend on the magnitude of τ.

A. Small delay

If the delay, τ, is small, then one may anticipate that it will have little effect on the system. In this situation, the following approach is commonly used in engineering/control theory:

1. Choose the k_j as if there were no delay, using standard control theory techniques.;

2. With the chosen k_j, determining the minimum delay, τ_d which causes instability.;

3. Check that $\tau < \tau_d$.

This is the approach taken in [13, 16]. We will refer to τ_d as the *destabilizing delay*.

B. Large delay

The time delays involved in the control of human balance are long [28, 29, 46]. In this case it is necessary to design the control by taking the delay into account. One way to do this is by analyzing the characteristic equation of the linearization of the model with the delayed feedback. For the full cart-pendulum model (3) with the feedback (11) this is:

$$\begin{aligned}\Delta(\lambda) \;=\; & \ell(m+4M)\lambda^4 + 4\ell\delta\lambda^3 - 3(m+M)g\lambda^2 - 3\delta g\lambda + \\ & e^{-\lambda\tau}((3k_4 - 4\ell k_3)\lambda^3 + (3k_2 - 4\ell k_1)\lambda^2 + 3k_3 g\lambda + 3k_1 g).\end{aligned} \tag{12}$$

This equation has the same form as (5); however some terms are modified because of the presence of the time delay. Thus the stability problem becomes that of determining, for a given set of the physical parameters M, m, δ, ℓ, g, how to choose the k_j so as to maximize the delay for which the upright position becomes unstable. To do this, one needs to determine how the stability of the upright equilibrium point depends on the choice of k_j as well as the time delay τ. Since this is a 5 parameter problem, a full analysis is difficult. A more tractable problem is to reduce the number of parameters to 3 (two of the k_j and the delay). This will give a characteristic equation that can be analyzed, but the result will not be optimal. One way of making this reduction is to decouple the dynamics of the cart from the pendulum by neglecting friction between the cart and the pendulum and taking $k_1 = k_3 = 0$ and $M + m \approx m$. An alternative is to choose two of the k_j's so that two of the necessary conditions for stability with zero delay are satisfied. The problem then becomes to determine the region of stability in terms of the other two k_j's and the delay. The former approach was taken by [6, 11] and the latter by [12]. Both analyses yielded similar results, which we now describe. For fixed values of the physical parameters, there exists a critical delay, τ_c, such that

1. If $\tau > \tau_c$ there are *no* control parameters that stabilize the pendulum in the upright position.

2. If $\tau < \tau_c$ there are *always* values of the control parameters that stabilize the pendulum in the upright position. The size of the set of control parameters that stabilize the pendulum *decreases* as the delay *increases*.

To illustrate these results consider the characteristic equation (12). Choosing

$$k_2 = \frac{4\ell}{3}k_3 + 5(m+M)g, \qquad k_4 = \frac{4\ell}{3}k_3, \tag{13}$$

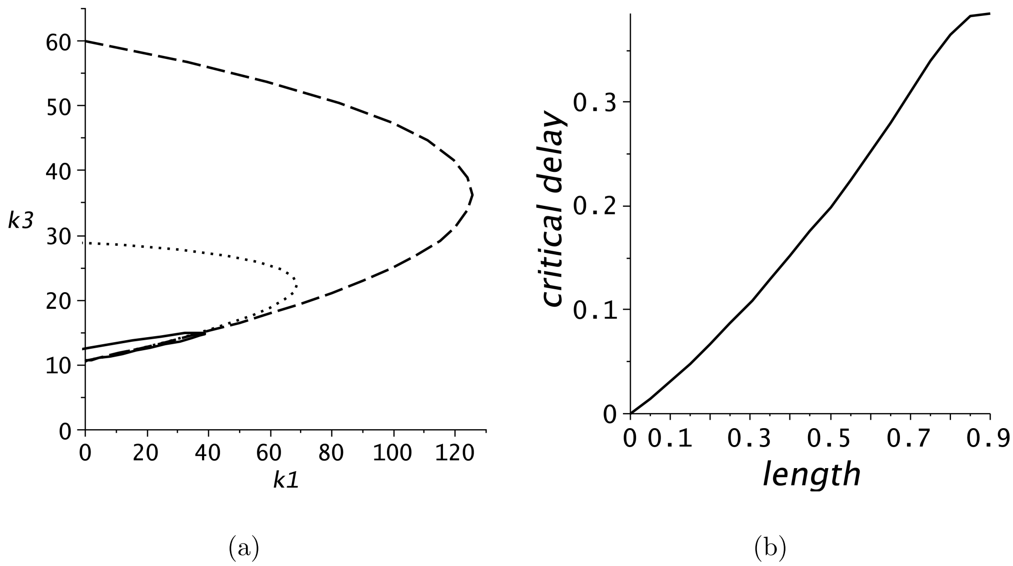


FIG. 2: Stability of the upright fixed–point for the model (3) with parameters corresponding to the experimental setup in [13] and k_2, k_4 chosen according to (13). (a) The stability region is terms of k_1, k_3 for $\tau = 0.01$ (dashed), $\tau = 0.05$ (dotted) and $\tau = 0.1$ (solid). (b) Effect of changing the length ℓ on the critical delay τ_c.

ensures that that conditions (7) are satisfied. Thus stability for $\tau = 0$ is guaranteed for any choice of k_1, k_3 satisfying (6). By analyzing (12) with $\tau \neq 0$ one can determine, for any τ sufficiently small, a region in the k_1, k_3 plane where the upright position is stable. As τ increases the region shrinks, until for $\tau = \tau_c$ it disappears entirely. These results are illustrated for the parameter values corresponding to the experimental setup of [13] in Figure 2(a). A similar illustration for (8) with delayed feedback given by (11) can be found in [11]. Stépán has also shown analytically [6, 11] that the critical delay for (8) is given by $\tau_c = \sqrt{\frac{2\ell}{3g}}$. Restating the analysis of [6, 11] for (10), shows that the critical delay for the torque control model is $\tau_c = \sqrt{\frac{8\ell}{3g}}$. These results show mathematically that the critical delay increases as the length increases, which is consistent with the experimental observation that long sticks are easier to balance at the fingertip than short ones. The corresponding analysis of (3) is more difficult, but a numerical investigation shows that τ_c increases as ℓ increases [12] (Figure 2(b)).

C. Two delays

For any real system it is possible to obtain instantaneous estimates of the force and displacement but not the velocity. Approximating speed requires that measurements be made at two distinct points in time, i.e.

$$\dot{\theta}(t) \sim \frac{\theta(t) - \theta(t-\tau_1)}{\tau_1} \tag{14}$$

where $\tau_1 > 0$ is the time interval, or delay, between the two measurements. Atay [9] has pursued this point in the context of a pendulum model similar to (10) where

$$T_{\text{control}} = T(\theta(t-\tau), \dot{\theta}(t-\tau_2))$$

where $\tau_2 = \tau + \tau_1$. Controllers of this form depend on the state at two different times and are sometimes referred to as *proportional minus delay* control [58]. When $\tau_2 = 2\tau$ Atay derived a result similar to those discussed above: for (10) there is a critical delay, $\tau_c = \sqrt{\frac{4\ell}{3g}}$, such that if $\tau < \tau_c$, then it is always possible to choose the parameters to stabilize the pendulum in the upright position.

D. Over–damping

A starting point for investigating the effects of the interplay between noise and delay is to reduce (1) to a first–order delay differential equation and assume that the effects of noise are additive, i.e. the effects of noise are independent of the state variable. Since postural sway mechanisms are likely to be over–damped in healthy individuals [29, 59], we have $\gamma\dot{\theta} \gg m\ell^2\ddot{\theta}$ and hence, for small θ, we have

$$\dot{\theta} - \gamma\theta + \sigma^2\xi(t) = f(\theta(t-\tau)) \tag{15}$$

where the additive gaussian white noise term, $\xi(t)$, satisfies

$$\begin{aligned} \langle \xi(t) \rangle &= 0, \\ \langle \xi(t)\xi'(t) \rangle &= \sigma^2\delta(t-t') \end{aligned}$$

where σ^2 is the variance and δ is the Dirac–delta function. Furthermore by taking into account the switch–like properties of the sensory and motor neurons involved in postural

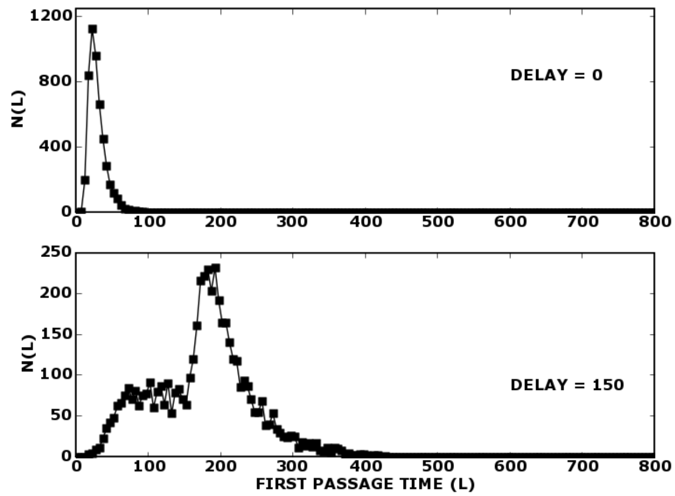


FIG. 3: The result of simulations of the first passage time distribution for a discretized equation $x(t+1) = x(t) + dt(\mu x(t-\tau) + \xi)$ where ξ is a gaussian white noise with variance σ^2. We have set the threshold at $X = 5.0$. The parameters are $dt = 1.0$, $\mu = 0.1$, $\sigma^2 = 0.3$ The statistics are averaged from 5000 realizations.

control [29, 32] we have

$$f(\theta(t-\tau)) = \begin{cases} 0 & \text{if } |\theta| \leq \Pi \\ -K & \text{otherwise} \end{cases} \tag{16}$$

This reduces the analysis of (15) to considerations of a first–passage time problem for an unstable fixed point (left–hand side of (15)) with re-injection into the interval $-\Pi \leq \theta \leq -\Pi$ wherever the threshold, Π is crossed.

Current interest has focused on the possibility that the left-hand side of (15) also contains a time delay. This gives rise to a unstable delayed random walk [32, 35, 38]. As is shown in Figure 3, the interplay between noise and delay can transiently stabilize the unstable fixed point, i.e. prolong the first passage time. These effects are interesting in light of measurements of the reaction time and response time when posture is perturbed [46]. In this study it was observed that the neural time delay, i.e. the time interval between the onset of a 3 cm postural displacement and the initiation of electromyographic activity, is $\sim$ 116ms (range 93–137ms depending on which muscle is recorded). However, the latency to reverse the perturbed movement is much longer, $\sim$ 320ms (range 177–492ms). Thus a passive control mechanisms that “fills in the gap” between the time the neural signal arrives

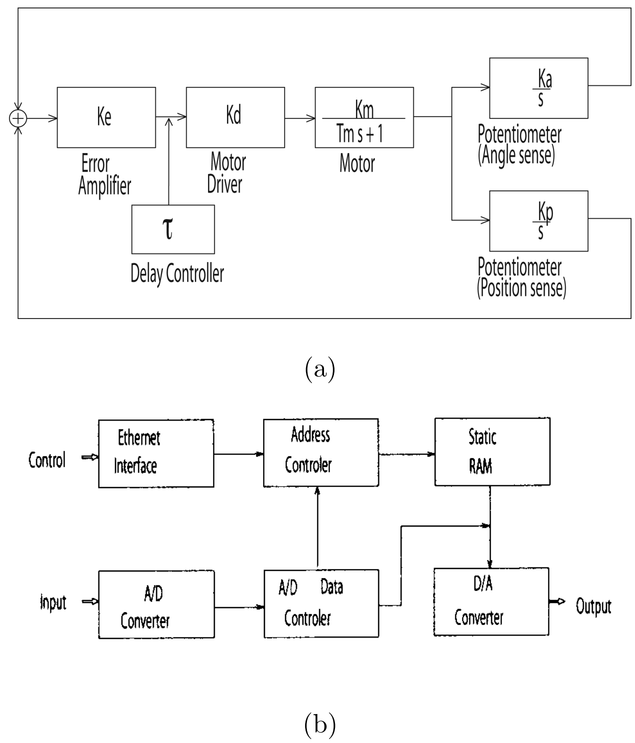


FIG. 4: Block diagrams for a) PID control and b) the delay control. For more details see [14].

at the neuro–muscular junction and the time to make a corrective movement would be useful for maintaining balance. This implies that passive control of this form can be part of the control of balance and, by implication, gait stability.

IV. METHODS

A. Delayed controller for inverted pendulum

We used a low friction time–delayed inverted pendulum controller that takes advantage of the properties of the carriage mechanism of DC–motor–operated plotters (Figure 1(b)). Previous implementations employing a mechanical cart are described in [13, 16]. Our system was designed to be capable of controlling both the vertical angle and x–position of the pendulum using separate proportional–integral–derivative (PID) controllers (see below) (Figure 4). The stick length was 0.39m and the track length was 0.29m. A potentiometer placed at the fulcrum of the pendulum detects the vertical displacement angle. A DC servomotor drives the slider on the rail using a timing belt, and the position of the slider is detected by a multi–rotational potentiometer. The timing belt compliance is very small

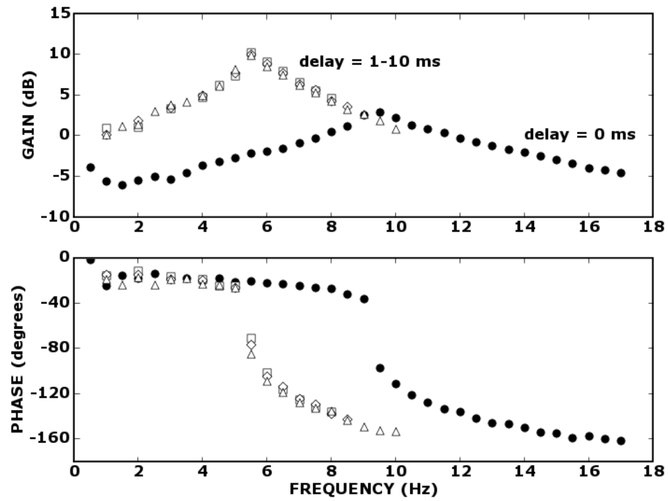


FIG. 5: The effects of changing the time delay on the transfer function of the PID controller: a) gain and b) phase.

and does not introduce unwanted poles within the bandwidth of the servomechanism. The error output signals of the PID controllers are added to produce the input signal for the motor driver. The DC motor is driven by a power amplifier similar to that used as an audio amplifier. The signal delay was introduced by first A/D converting it and writing it to a static RAM (Figure 4(b). The contents of the RAM were read out after a specified time, τ, and then D/A converted to produce an output signal. The delay time, τ, was controlled by an outside PC using the Ethernet. The current sampling period is 1ms, the maximum signal delay is approximately 4s, and the granularity of the control is 1ms.

We used PID controller to regulate the angle of the stick and the position of the cart [60]. A PID controller is a three–term feedback controller: the P component is proportional to the error, i.e. the difference between the current angle and the target angle of the stick or the current and target position of the cart, the I component is proportional to the integral of the error over some time interval, and the D component is proportional to the derivative of the error. In our case the P component greatly reduced the error; however, because of inertial effects the error could not be reduced to zero. Therefore we included an I component to make the error zero: by summing over a long enough time interval even a small error can produce a big enough drive signal to reduce the error. Finally the D component, which does not effect the error, was adjusted to minimize overshot. Figure 5 shows the open–loop transfer function of the delayed pendulum controller with and without delay. A time delay is not expected to effect the gain of the transfer function, but adds a contribution $-f\tau$ to the phase, where f is the frequency. When $\tau = 0$, the amplitude of the transfer function has a peak of about 3 db at ~ 9.5 Hz which is related to the damping ratio of the second–order transfer function. When $\tau \neq 0$ this peak increased in magnitude and was shifted to a lower frequency, suggesting that the response of the PID controller is limited by its slew rate (proportional to frequency times the gain). However, over the range of delay between $1 - 10$ms the slew rate was approximately constant and did not itself affect the stability of the delayed pendulum controller.

B. Stick balancing at the fingertip

Stick balancing was performed while the subject was seated comfortably in a chair as described previously [20, 21]. The subjects, ages 18–58 years, were required to keep their back in contact with the chair at all times with their arm extended in front of them. In this position the subject could not see both the position of the tip of the stick and the fingertip at the same time in their field of view. Sticks were wooden dowels with diameter 6.35 mm and length ~ 0.55m (i.e. $\ell = .275$m). Reflective markers were attached to each end of the stick and three specialized motion cameras (Qualisys Oqus, Model 300) detected infrared light reflected from these markers. The image detected by each camera determines two of the spatial coordinates: the third coordinate is determined by triangulation methods involving at least two of the cameras. Subjects reported in this communication had moderate skill levels had increased their stick balancing skill with practice by about 2–fold (typically from a mean survival time of $8 - 12$s to $17 - 25$s for 25 consecutive trials).

We calculated the change in speed of the fingertip, ΔV_f, using the bottom marker attached to the stick as follows [21]: The change in the position of the marker, $\Delta\vec{r}(t)$, in one time step Δt is $\Delta\vec{r}(t) = \vec{r}(t + \Delta t) - \vec{r}(t)$, where the notation $\vec{r}$ denotes the position vector. All vectors were measured from a common reference point provided by the Qualisys measurement system. The magnitude of the mean speed, V, is

$$V(t) = \left\| \frac{\Delta\vec{r}(t)}{\Delta t} \right\|$$

where the notation $\|.\|$ denotes the norm, and hence

$$\Delta V_f(t) = V(t + \Delta t) - V(t) \tag{17}$$

C. Human postural sway

Measurements of the center of pressure (COP) were obtained by having subjects stand in stocking feet on a pressure platform (Accusway, AMTI). Subjects were asked to look straight away with eyes closed while remaining as still as possible. The sampling frequency was 200 Hz and the data was re–sampled at 100 Hz. For postural sway $K(s)$ was calculated using only the displacements in th anterior–psoterior (AP), y, and medio–lateral (ML), x, directions.

We analyzed the fluctuations in COP in the context of a correlated random walk [26, 27, 32]. The two–point correlation function, $K(s)$, was calculated as [32]

$$K(s) = \frac{1}{N-n} \sum_{i=1}^{N-n} [(x(t_i) - x(t_i + s))^2 + (y(t_i) - y(t_i + s))^2] \tag{18}$$

For each $s = |t_1 - t_2|$, the two–point correlations are calculated from N data points spanning $N-n$ data intervals of length ns and where x indicates the displacements of the fluctuations in the anterior–posterior direction, and y the displacements in the medial-lateral direction. For a correlated random walk [26, 27]

$$K(s) \sim s^{2H}$$

where H is a scaling factor such that $H > 0.5$ indicates positive correlation (*persistence*) and $H < 0.5$ indicates negative correlation (*anti–persistence*). For stick balancing we calculated $K(s)$ for the movements of the fingertip in the same way except the fluctuations in the vertical direction, z, were also included.

All of the experiments involving human subjects were performed according to the principles of the Declaration of Helsinki and informed consent was obtained. Experimental protocols for human postural sway and stick balancing at the fingertip received separate approval by the institutional review board at Claremont McKenna College.

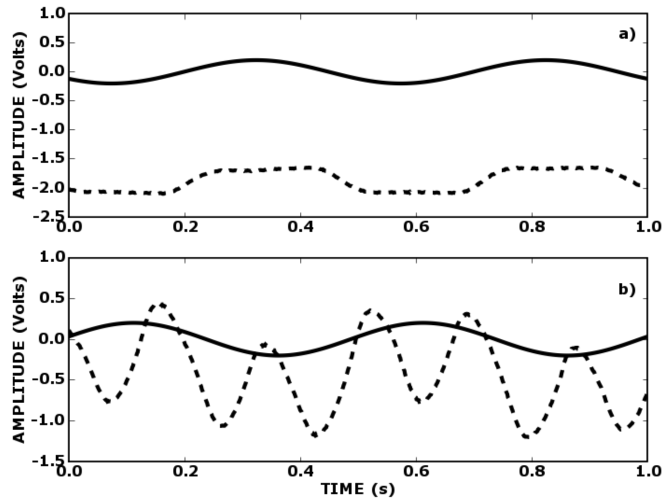


FIG. 6: Response of the delay controller (dashed line) to a 2 Hz input input frequency (solid line) for different time delays: a) 5 ms and b) 15 ms. See [14] for more details.

V. RESULTS

A. Mechanical stick balancing

We first examined the behavior when the PID controller related to the x–position was omitted. Since the transfer function is known (Figure 5), we can determine the dynamics by simply injecting sinusoidal inputs. From this perspective, stability of the upright fixed–point means that the input and response frequencies are the same, and instability that the frequencies are different. Figure 6 shows that the delayed inverted pendulum controller exhibits two behaviors depending on the choice of the delay and frequency of the input. Instability of the upright position was characterized by a difference between the input and response frequencies. For example, when $f = 2$Hz we have stability when $\tau = 5$ ms (Figure 6a) and closed–loop instability, i.e. "hunting", when $\tau = 15$ms (Figure 6b). However, for $\tau = 15$ms we observed that stability could be achieved by increasing the input frequency to $4 - 10$ Hz. If we take $\tau_{\rm c} = \sqrt{\frac{2\ell}{3g}} \sim 115$ms, then for these delays the upright fixed–point can be stable. However, we observed that even in the "hunting" regime the stick remained upright albeit with oscillatory dynamics.

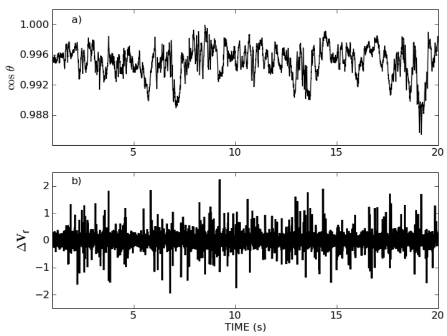
We next examined the behavior of the time-delayed inverted pendulum controller when both the PID controller for the angle and position were activated. The I loop of the angle–

PID is absolutely necessary to balance the inverted pendulum since the average error to the right and left is zero only at the balanced angle. This occurs when the control works to make the angle–PID integration error zero. However, the PID–distance controller (negative feedback) for the position stabilizer of the slider functions like a positive feedback for the inverted pendulum and vice versa. In other words, whenever we increase the slider position error so that the position shift is effective in activating the PID–distance controller, we necessarily destabilize the PID–angle controller. On the basis of these experimental results we conclude that we cannot control both the vertical angle and the position of an inverted pendulum, at least when using PID–controllers restricted to the horizontal plane (see DISCUSSION).

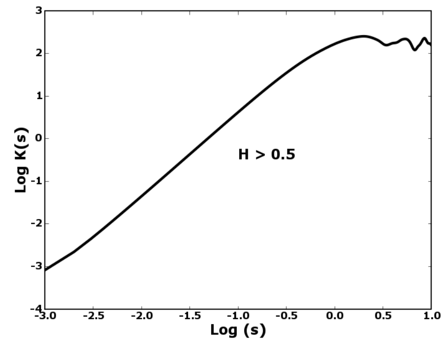
B. Human stick balancing

For stick balancing at the fingertip, there are two ways the stick can fall, and hence, as for mechanical stick balancing, two control problems: 1) the vertical displacement angle, θ, becomes too large; and 2) the position of the hand drifts out of reach of the arm. Our focus here on the first control problem and, in particular, on the nature of the control that occurs on time scales equal to or less than the neural latency [20, 25]. Figure 7(a) compares the movements of the vertical displacement angle, θ, calculated as $\Delta z/\ell$, to the changes in speed, ΔV_R, made by the fingertip. Clearly the relationship between controlled variable (θ) and controller (ΔV_f) is very different than seen for mechanical stick balancing (Figure 7(a)). Whereas for mechanical stick balancing the controlling forces vary sinusoidally, those for stick balancing occur intermittently. Indeed it has been shown that the times between successive corrective (upward) movements obey a $-3/2$–power law [20]. Power laws with this exponent can be accounted for by assuming that one of the control parameters is stochastically forced back and forth across a stability boundary [20]. In other words the balance control system is tuned near or at the "edge of stability". This interpretation is consistent with the observation that $\tau_{\rm n} \sim \tau_{\rm c} = \sqrt{\frac{2\ell}{3g}} \sim 140$ms where $\tau_{\rm n}$ is estimated using the cross–correlation between the movements of the fingertip and tip of stick (estimates of $\tau_{\rm n}$ using different techniques yield larger values [19, 24].) An alternate interpretation is that these power laws arise because of a time-delayed optimal control mechanism [61].

Figure 7(b) shows the two–point correlation function for the movements of the fingertip. For small displacements $H > 0.5$ (observed for 9 subjects) and hence there is persistence.



(a)



(b)

FIG. 7: Dynamics of stick balancing at the fingertip: (a) compares the time series of the cosine of vertical displacement angle, equal to $\Delta z/\ell$, to that of the changes in speed, ΔV_f of the fingertip, and (b) shows $K(s)$ for the movements of the fingertip (lower marker on stick) calculated using (18). The digitization rate is 1000 Hz. For this subject $H \sim 0.72$.

The simplest interpretation of this observation is that the upright fixed–point is unstable and hence sufficiently close to this fixed–point the system is allowed to drift away. Indeed it has been suggested that for a system at the edge of stability, the fluctuations resemble a delayed random walk whose mean displacement is approximately zero [20]. For the mechanical inverted pendulum, the upright fixed–point in the "hunting" regime is also unstable even though the stick remains upright. However, in this case the dynamics of the controller become clearly oscillatory. For stick balancing it is clear that the behavior of ΔV_f is more complex.

C. Human postural sway

Two concepts are important for understanding the control of human balance during quiet standing [30, 31]: 1) center of mass (COM), the net location of the center of mass in 3–D space, and 2) center of pressure (COP), the weighted average of the location of all downward (action) forces acting on the standing surface. Typically, COM is computed by making a weighted average of the COM's of each body segment using a total body model [30, 31], whereas COP is measured using a force platform [31]. The COP represents the neuromuscular response to imbalances of the body's COM, i.e. when the COM is displaced from the neutral axis of alignment, compensatory changes must be made in COP to re-direct the COM back toward the neutral axis. These compensatory changes are related to neuromuscular forces. Previous studies have shown that on slow time scales (digitization rate 20 Hz) COP regularly oscillates about COM in the AP–direction; however, more complex behaviors are seen in the ML–direction [30, 31]

Figure 8a shows the fluctuations of the COP in the (x, y) plane for a single subject. For slightly less than one–third of subjects, $K(s)$ could be described by three scaling regions demarcated by the ↓ in Figure 8b as described previously [26, 27]. However, for other subjects $K(s)$ could not be represented by three scaling regions [28, 32]. Of these subjects, two patterns could be distinguished, an oscillatory $K(s)$ and a non–oscillatory $K(s)$ (Figure 8c). In all cases, for small displacements we observed that $H > 0.5$ and for large displacements $H < 0.5$. The fact that the difference types of $K(s)$ could be observed in the subject, recorded at different times, suggests that the variations in $K(s)$ have a dynamic basis. This interpretation in supported by the fact that all patterns could be reproduced by a simple model for postural sway, namely (15)–(16), by varying the noise intensity [29, 32] (Figure 8d). The observation that the upright fixed–point for postural sway is unstable is consistent with the measured latencies. The COM for a standing human is located approximately at the level of the second sacral vertebrae, i.e. $\ell \sim$ 1m from the standing surface. This gives $\tau_{\mathrm{c}} = \sqrt{\frac{2\ell}{3g}} =$ 260ms. Thus τ_c is shorter that the neural time delay, but longer that the time delay to reverse the perturbed movement τ_n (Section III D).

Typically the COP fluctuations are slightly biased in the AP–direction (as shown); however, some subjects the COP fluctuations are not biased or slightly biased to the left or right. There was no relationship between the bias in the COP fluctuations and the type of $K(s)$ pattern observed.

VI. DISCUSSION

Our observations demonstrate that for three paradigms of human balance control, namely mechanical stick balancing, human stick balancing at the fingertip and postural sway during quiet standing, the fixed–point for the upright position is unstable. This conclusion is supported by direct comparisons of the movements of the inverted pendulum and the controller and, in the case of human balance control, the fact that $H > 0.5$ for small displacements. Mathematical studies of time-delayed feedback control emphasize the importance of mea-

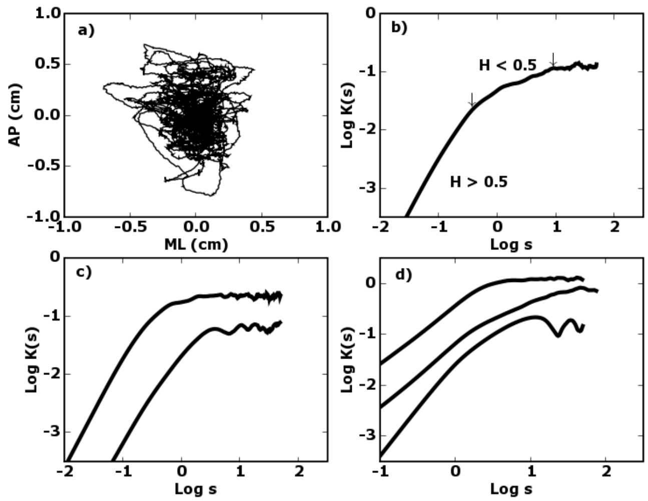


FIG. 8: Dynamics of the fluctuations in the center of pressure (COP) measured during quiet standing with eyes closed using a force platform. (a) Trajectories of COP projected onto the anterior–posterior (AP): medial–lateral (ML) plane.; (b) Comparison of the project area of the fluctuations in COP to the area of the base of support, i.e. the sum of the area under and between the subject's feet.; (b)-(c) show different patterns of $K(s)$, and (d) shows that the different patterns of $K(s)$ can be generated from (15)–(16) by changing the noise intensity. The $K(s)$ are arranged with the lowest noise intensity on the bottom and the highest on the top. See text for discussion.

suring the relative magnitudes of τ_c and τ_n. However, there are several problems associated with making decisions about stability based solely on measurements of these delays. First, although $\tau_n > \tau_c$ guarantees instability of the fixed–point, $\tau_n < \tau_c$ does not guarantee stability. Second, it is difficult to apply these criteria to human data since estimates of τ_n vary depending on how you measure them (see Sections VB and VC). Finally, and more importantly, focusing on τ_n and τ_c overlooks the fact that instability can arise simply because of the inherent difficulties of simultaneously controlling the position of the inverted pendulum and the controller using delayed feedbacks. In other words the balanced state is stabilizable, but not controllable (see Section II). Several empirical observations support this issue as a fundamental mechanism for balance instability: our inability to control a mechanical inverted pendulum with two PIDs controllers, published time series of COM and COP for postural sway [30, 31] and the observed continual movements of the hand of even an expert stick balancer.

Currently it is believed that a better way to view the balanced state is as a state in which the vertical displacement angle is confined, or bounded, in some manner within an acceptable range about $\theta = 0$ [20, 26, 35, 54]. One way that this can be accomplished is through the appearance of bounded, time dependent oscillatory types of attractors, e.g. limit cycle, quasiperiodic, chaotic, and so on. It is well established that feedback control with delay can readily generated these behaviors through both supercritical and subcritical Hopf bifurcations [15, 16, 51]. The 'hunting' behavior observed for mechanical inverted pendulums and the COP oscillations about COM recorded in the AP–direction for human postural sway [30, 31] suggests that oscillatory types of attractors may be part of the solution. However, there are a number of reasons to believe that the approach taken by the nervous system to control human balance may be fundamentally different that the approaches typically taken by engineers to stabilize a mechanical inverted pendulum. We discuss our reasoning in terms of four additional misconceptions that arise in biomechanical discussions of gait and postural stability when considerations of time delays are omitted.

First, in the application of control engineering concepts to the nervous system it is often implicitly assumed that neural feedback operates continuously. Putting aside considerations of the high costs associated with implementing such strategies, the main problem is that continuous feedback is not desirable for stabilizing an unstable fixed point in the presence of noise and delay [33–35]. The problem is distinguishing those fluctuations that need to be acted upon by the controller from those that do not. This is because, by definition, there is a finite probability that an initial deviation away from the set point will be counter–balanced by one towards the set point just by chance. Too quick a response by the controller to a given deviation can lead to "over control" leading to destabilization, particularly when time delays are appreciable. On the other hand, waiting too long runs the risk that the control may be applied too late to be effective. Thus methods based on continuous feedback are not only anticipated to be very difficult to implement by the nervous system, but are also unlikely to be effective. One way to achieve effective control in the presence of noise and delay is to use an "act–and–wait" type of control strategy [33, 34]. An act–and–wait control strategy is a type of adaptive control in which when a corrective force is generated ('act') it is necessary to 'wait' sometime before the next corrective force is generated. One possible way to implement an "act-and–wait" control strategy is to use a switch–like controller, in which corrective outputs are generated only when the dynamical variables cross pre–set thresholds

[32, 35, 54]. Switch–like adaptive controllers are well known to engineers and have the property that they are optimal when the control is bounded [4]. The intermittent controlling movements observed for both stick balancing at the fingertip (Figure 7(a) and [20]) are certainly consistent with the notion of discontinuous control. In retrospect, measurements of the two–point correlation function for human postural sway were the first to draw attention to the possibility of a act–and–wait control strategy for balance control [26, 27]. Finally the existence of an adaptive type of controller for postural balance might explain the observation that although balance instability increases in those elderly subjects who have a prolonged τ_n, these subjects nonetheless remain upright most of the time [62]!

The second misconception that has arisen in biomechanical discussions of gait and posture stability as a consequence of neglecting the importance of time delays is the tendency to equate oscillations with the notion of passive feedback, i.e. feedback that relies solely on the biomechanical properties of joints and their associated connective tissues (see, for example, [30, 31]). Indeed the aforementioned oscillations of COP about COM in the AP–direction during postural sway were initially interpreted in terms of a harmonic oscillator–type model [30]. This interpretation led to two untenable additional assumptions, namely, 1) damping was precisely zero (not true [29, 59]), and 2) balance control during quiet standing was entirely maintained by the biomechanical stiffness of the ankle joint (also not completely true [56, 57]). In contrast, stable limit cycle oscillations readily arise in models of delayed inverted pendulums even when they are damped, either because the feedback is switch–like [29] or because the destabilizing delay is exceeded and hence the equilibrium point becomes unstable [13, 15, 16, 51]. Thus there is no reason to ignore the effect of damping to account for the oscillations observed in balance control or, even to assume that the presence of oscillations eliminates the possibility of active neural feedback control.

A third misconception concerns whether it is possible to control simultaneously both the angle and the pivot point at an arbitrary position of the pendulum using linear feedback. The observations in Section II suggest that this is not possible when $\tau = 0$. Our observations suggest that this cannot be achieved when $\tau \neq 0$, at least by using PID–type controllers. This is another reason why the dynamics of human balance control are so complex (see, for example, [20, 30, 52]. A closely related issue concerns how the nervous system estimates speed (derivative) of moving object since speed is included in the feedback controllers used by engineers to stabilize the pendulum's upright position. In order to measure a speed it is necessary to obtain measurements at two points in time. Equation (14) implies that there is likely to be an intimate relationship between the fact that the nervous system is constructed of delay lines and the estimation of spatial and temporal derivatives. Certainly the visual system has the ability to estimate speed of moving objects [63] and indeed it has been possible to construct a silicon retina that measures speed by incorporating features that mimic those of neurons in the retina [64, 65]. However, it is not known whether this can also be accomplished by using the non–visual nervous system with sufficient accuracy to enable an inverted pendulum to be stabilized. This observation may explain why it is much easier to balance a light stick at the fingertip with eyes open, than with eyes closed. Along these lines we might speculate that the continued movements of the hand (and hence fingertip) in the horizontal plane of even a very expert stick balancer arise because the nervous system has access only to poor information regarding the velocity of hand movements which are not normally located within the visual field of the balancer during the performance of this task (see METHODS). Moreover it becomes less clear whether changes observed in gait width are a stabilizing mechanism [66–68], or simply a reflection of the inability of the nervous system to simultaneously control both gait width and vertical stability.

A final misconception is the belief that random perturbations ("noise") have only deleterious effects on balance control. It is important for the physically–oriented reader to note that neuroscientists working on human balance control typically use the term "noise" to refer to either the noise–like components of muscle activity [48] or to externally generated vibratory inputs applied to the body [69]. It is becoming clear that these types of noisy inputs can have beneficial effects on balance control. For example, vibrations applied to the soles of the feet can stabilize postural sway through the ability of sub-threshold vibrations to enhance the sensitivity of relevant sensory neurons via a mechanism known as "stochastic resonance" [69]. Recently attention has focused on the possibility that noise can directly confine an unstable dynamical system close to the origin in the presence of retarded variables [35–38]. Thus the observation that postural sway in the elderly is characterized by both increased muscle activity [48] and the use of open–loop control for longer time intervals [62] may be a consequence of an increased reliance on passive control mechanisms that arise from the interplay between noise and delay.

Evaluating control strategies for real dynamical systems requires careful consideration as to whether it is feasible to implement the strategy given the inherent limitations of the resources at hand. Control strategies that involve measurements of displacement and velocity are useful for mechanical systems, e.g. feedback and feed–forward control, when the time interval required for the estimation of the velocity can be made sufficiently short; though even here problems exist [7, 8, 10]. Although these engineering concepts have heavily invaded the neuroscience literature, it is completely unclear whether the nervous system attempts the same types of control that engineers attempt to implement. The nervous system may take advantage, in some way, of the long delays that are present to use novel and perhaps more robust control strategies (see also [70]). Near the edge of stability, stochastic forms of control become possible that depend on the interplay between noise and delay [20, 35, 36]. Perhaps the nervous system uses adaptive "act and wait" control strategies simply because they are cheaper to implement and maintain. In any case, until issues such as these are resolved, we suggest that conclusions drawn from the application of control engineering concepts to the nervous system be interpreted cautiously.

VII. ACKNOWLEDGMENT

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