The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
and
The Canadian Mathematics Competition
presents

The First Annual Small c Competition

for First Year Students

Saturday 29 September 2001

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open the contest booklet until you are told to do so.
2. You may use slide rules, abaci, rulers, protractors, compasses and paper for rough work.
3. On your response form, print your name, program, and ID number.
4. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, enter it in the appropriate box on the response form.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
6. Diagrams are *not* drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A

1. The value of $3^0 - 2^1 + 1^2 - 0^3$ is

   (A) 4 **(B)** 2 **(C)** 0 **(D)** 3 **(E)** 1

2. Christopher has written five math tests. He obtained marks of 70%, 82%, 80%, 65%, and 85%. What mark must he get on the next test to have an overall average of 80%, assuming that all six tests are equally weighted?

   (A) 78% **(B)** 80% **(C)** 88% **(D)** 98% **(E)** 100%

3. If $a\bigstar b = a + b^2$, then $(2\bigstar 3)\bigstar 4$ equals

   (A) 27 **(B)** 9 **(C)** 24 **(D)** 53 **(E)** 23

4. The points $A(4, -3)$, $B(7, -8)$ and $C(w, w)$ all lie on the same straight line. The value of w is

   (A) $\frac{2}{11}$ **(B)** 1 **(C)** $\frac{11}{8}$ **(D)** 0 **(E)** $\frac{11}{6}$

5. When $\frac{1}{4}$ of a number is subtracted from $\frac{1}{3}$ of the same number, the result is 48. The original number is

   (A) 4 **(B)** 12 **(C)** 144 **(D)** 576 **(E)** 1152

6. In degrees, the equivalent of $\dfrac{7\pi}{18}$ radians is

   (A) 430 **(B)** 140 **(C)** 1.22 **(D)** 35 **(E)** 70

7. The circumference of a circle is 40 cm. If the circumference is increased by 2 cm, then the increase, in cm, of the radius is

   (A) π **(B)** 1 **(C)** 2 **(D)** $\dfrac{1}{\pi}$ **(E)** $\dfrac{2}{\pi}$

8. The squares of the first four prime numbers are added together. The largest prime number which divides evenly into this sum is

   (A) 19 **(B)** 29 **(C)** 3 **(D)** 7 **(E)** 17

9. Two fair dice are rolled. The probability that the sum of the numbers rolled is divisible by 3 is

   (A) $\frac{4}{11}$ **(B)** $\frac{11}{36}$ **(C)** $\frac{1}{3}$ **(D)** 2 **(E)** $\frac{1}{9}$

10. Albert's watch is 2 minutes slow and Matilda's is 3 minutes fast (relative to a National Time Signal). Matilda thinks that her watch shows the right time, but Albert thinks that his is 4 minutes fast. They are supposed to meet at the train station at 7:55 a.m. because the train comes in at 7:57 a.m. and leaves at 8:00 a.m. What will probably happen is that

    (A) Albert will miss the train.
    (B) Matilda will miss the train.
    (C) Albert will get on the train before Matilda.
    (D) They will both miss the train.
    (E) They will get on the train together.

Part B

11. If $a+b=12$, $b+c=16$ and $c+a=14$, the value of b is

(A) 8 **(B)** 7 **(C)** 5 **(D)** 9 **(E)** 4

12. If $x-y=xy=\frac{x}{y}$ with x and y real numbers, the value of x^2+y^2 is

(A) $\frac{5}{4}$ **(B)** $\frac{17}{4}$ **(C)** 0 **(D)** 2 **(E)** $\frac{1}{2}$

13. Eight different numbers are chosen from the set $\{1,2,3,4,5,6,7,8,9,10\}$ and are then added up. Which of the following sums is impossible?

(A) 52 **(B)** 44 **(C)** 41 **(D)** 38 **(E)** 35

14. The minimum value of $f(x)=x^4-4x^3+6$ for $1\le x\le 4$ is

(A) -75 **(B)** -21 **(C)** 1 **(D)** 3 **(E)** 6

15. The numerical value of $\lim\limits_{x\to 2}\dfrac{x-2}{2+3x-2x^2}$ is

(A) $-\frac{1}{3}$ **(B)** $\frac{1}{3}$ **(C)** ∞ **(D)** 0 **(E)** $-\frac{1}{5}$

16. A 26 metre ladder leans against a building so that its foot moves away from the building at a rate of 3 m/s. When the foot of the ladder is 10 metres from the building, the top is moving down the wall at a rate of r m/s, where r is

(A) $-\frac{5}{4}$ **(B)** $-\frac{1}{3}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{5}{4}$ **(E)** 3

17. The curve in the xy-plane defined by the equation $\left(x^2-1\right)y=x^2+1$ has

(A) 1 horizontal asymptote and 1 vertical asymptote.
(B) 0 horizontal asymptotes and 2 vertical asymptotes.
(C) 1 horizontal asymptote and 2 vertical asymptotes.
(D) 2 horizontal asymptotes and 2 vertical asymptotes.
(E) 0 horizontal asymptotes and 0 vertical asymptotes.

18. The number of solutions of the equation $\sin^2 x=\cos x$ with $0<x<2\pi$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** π **(E)** 4

19. The number of three-digit positive integers whose digits have a product of 24 is

(A) 4 **(B)** 30 **(C)** 18 **(D)** 21 **(E)** 24

20. What is the maximum value of $x+y$, given that $x^2+2y^2=60$?

(A) $3\sqrt{10}$ **(B)** $10\sqrt{3}$ **(C)** 10 **(D)** $2+2\sqrt{7}$ **(E)** $6+2\sqrt{3}$

Part C

21. In the diagram, $PBCQ$ is a trapezoid with $PQ = 2$ and $BC = 3$. If the area of triangle ABC is 36, then the area of $PBCQ$ is

(A) 90 **(B)** 95 **(C)** 110 **(D)** 100 **(E)** 105

22. Starting at the point $P(x, y)$ on the coordinate plane, a pin is randomly moved to either the point $A(x + 1, y)$ or the point $B(x, y + 1)$, with each move having a probability of $\frac{1}{2}$. If the pin starts at $(0, 0)$ and is moved to $(4, 4)$, what is the probability that it passes through $(2, 2)$?

(A) $\frac{6}{35}$ **(B)** $\frac{12}{35}$ **(C)** $\frac{16}{35}$ **(D)** $\frac{18}{35}$ **(E)** $\frac{8}{35}$

23. In an arithmetic sequence, the sum of the first $2n$ terms is equal to the sum of the next n terms, $n \neq 0$. If the first term is 2 005 003 and the common difference is n, then the value of n is

(A) 2000 **(B)** 2004 **(C)** 2001 **(D)** 2003 **(E)** 2002

24. A circle of radius 1 is divided into 3 equal arcs which are combined as shown in the diagram. The area enclosed by the 3 petals is

(A) $\pi - \frac{3\sqrt{3}}{2}$ **(B)** $\pi - \frac{3\sqrt{3}}{4}$ **(C)** $2\pi - 3\sqrt{3}$ **(D)** $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ **(E)** $\frac{\pi}{3}$

25. If a, b, c are the roots of $x^3 + 2x^2 + 7x = 19$, then the value of $a^3 + b^3 + c^3$ is

(A) 89 **(B)** 91 **(C)** 93 **(D)** 95 **(E)** 97