The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
and
The Canadian Mathematics Competition
presents

The Second Annual Small c Competition

for First and Second Year Students

Saturday 05 October 2002

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open the contest booklet until you are told to do so.
2. You may use slide rules, abaci, rulers, protractors, compasses and paper for rough work. You may also use log tables; log cabins are not permitted.
3. On your response form, print your name, program (plan?), and ID number.
4. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, enter it in the appropriate box on the response form.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
6. Diagrams are *not* drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A

1. The value of $1 + 2 \times 3 + 4^2 - 5$ is

   (A) 17 **(B)** 18 **(C)** 20 **(D)** 42 **(E)** 52

2. Conrad's MATH 235 class was scheduled to make group presentations. When Alan's group was presenting, he noticed that $\frac{1}{6}$ of the class had already made their presentations, and $\frac{2}{3}$ of the class still had theirs to do. If each group has 4 members, how many students were in the class?

   (A) 12 **(B)** 17 **(C)** 24 **(D)** 36 **(E)** 48

3. $\sqrt{7 + 4\sqrt{3}}$ is equal to

   (A) $3 - \sqrt{2}$ **(B)** $\sqrt{14}$ **(C)** $2 + \sqrt{3}$ **(D)** $2\sqrt{3}$ **(E)** $1 + 2\sqrt{3}$

4. The sequence of numbers $a_1, a_2, a_3, \ldots$ satisfies the relation $\dfrac{a_m}{a_n} = \dfrac{m}{n}$ for every pair of positive integers m and n. If $a_3 = 5$, the value of $3a_{20}$ is

   (A) 20 **(B)** 33 **(C)** 90 **(D)** 100 **(E)** 300

5. Fred (from Flin Flon) flips five fair French francs. (A franc was a French coin before the Euro became the official currency.) What is the probability that there are at least three heads showing after the coins are flipped?

   (A) $\frac{3}{32}$ **(B)** $\frac{5}{32}$ **(C)** $\frac{15}{32}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{15}{16}$

6. Eight congruent pieces of paper are stacked, as shown in the view from above. The order in which they were laid down (first to last) was

   (A) CDHGFEBA **(B)** GFEHCDBA
   (C) GCHDFEBA **(D)** CHGFDEBA
   (E) HGFEDCBA

7. The "C & D" virus is eating the memory on a hard drive. The first day, it eats half the memory, the second day it eats one third of the remaining space, the third day it eats one quarter of the remaining space, and on the fourth day it eats one fifth of the remaining space. What fraction of the original memory is left after the fourth day?

   (A) $\frac{1}{5}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{24}$

8. A solid sphere of uniform density has a radius of 15 cm and weighs 135 kg. Another sphere of the same material weighs 40 kg. If the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$, the radius of the second sphere, in centimetres, is

   (A) 5 **(B)** 10 **(C)** 15 **(D)** $\sqrt[3]{\frac{5400}{\pi}}$ **(E)** 20

9. The largest integer n for which $n^{2002} < 5^{3003}$ is

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 11 **(E)** 12

10. A "super-kangaroo" hops from Melbourne to Ayers Rock, a distance of 2095 km. If its first hop is 1 m long, and each hop is twice as long as the previous hop, how many hops will it take to get to Ayers Rock?

    (A) 20 **(B)** 19 **(C)** 22 **(D)** 21 **(E)** 2 095 000

Part B

11. A pirate ship had enough food to feed its crew for 60 days when it picked up 30 shipwrecked sailors. Immediately after the pick-up, the food on board was then enough to feed the crew plus the shipwrecked sailors for 50 days. How many crew members did the pirate ship have?

    (A) 15 **(B)** 40 **(C)** 110 **(D)** 140 **(E)** 150

12. A large rectangle is divided into four identical rectangles, as shown. A pair of lines is to be drawn which will then divide the large rectangle into eight equal areas. How many possible pairs of lines will do this?

    (A) 3 **(B)** 5
    (C) 6 **(D)** 10
    (E) Infinitely many

13. The straight line with equation $y = x + 2$ intersects the parabola $y = x^2$ at two points A and B. If M is the midpoint of A and B, and C is the vertex of the parabola, the equation of the line passing through C and M is

    (A) $y = 5x$ **(B)** $x = 0$ **(C)** $y = 3x$ **(D)** $x - y = 2$ **(E)** $y = x^2 + x + 2$

14. The maximum value of the sum of the digits of the sum of the digits of a three digit integer is

    (A) 11 **(B)** 10 **(C)** 9 **(D)** 8 **(E)** 7

15. The number of integers x for which $\sqrt{9+x} + \sqrt{16-x}$ is also an integer, is

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 8

16. In the diagram, the three squares have side lengths 4, 7 and x, respectively, and the points A, B and C lie on a straight line. The value of x is

    (A) 10 **(B)** $\frac{49}{4}$
    (C) 11 **(D)** 13
    (E) $\frac{21}{4}$

17. Given a real number x, a robot can compute each of the four quantities: $x + 3$, $x - 2$, x^{-1} and x^2. If y is the largest real number that the robot can achieve after 3 successive operations starting from $x = 1.99$, then

    (A) $y = (1.99)^8$ **(B)** $y = (4.99)^4$ **(C)** $y = (7.99)^2$ **(D)** $y > 20\,000$ **(E)** $1000 < y < 20\,000$

18. An *emirp* is a prime number that remains a prime when its digits are reversed. For example, 13 and 31 are two emirps since 13 and 31 are both prime. The number of two-digit emirps is

    (A) 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

19. The line with equations $\dfrac{x-4}{2} = \dfrac{y+5}{3} = \dfrac{z}{-2}$ intersects the xy-plane at the point A, and the yz-plane at the point B. The length of the line segment AB is

    (A) 4 **(B)** 5 **(C)** $\frac{5}{3}\sqrt{13}$ **(D)** $3\sqrt{5}$ **(E)** $2\sqrt{17}$

20. The sum of all two digit positive integers which give a remainder of 4 when divided by 5 is

    (A) 1030 **(B)** 1017 **(C)** 2034 **(D)** 486 **(E)** 1016

Part C

21. An archway is constructed on a straight base PQ with 2 circular arcs PR and QR. Arc PR has centre Q and arc QR has centre P. If the length of PQ is 2 m, then the area under the archway, in m^2, is

(A) $\frac{2}{3}\pi - \sqrt{3}$ **(B)** $\frac{1}{3}\pi - \frac{1}{3}\sqrt{3}$
(C) $\frac{1}{6}\pi + \sqrt{3}$ **(D)** $\frac{1}{2}\pi + \frac{1}{2}\sqrt{3}$
(E) $\frac{4}{3}\pi - \sqrt{3}$

22. The minimum value of $2x^2 + 2xy + y^2 - 8x - 2y + 13$, where x and y are real numbers, is

(A) $\frac{5}{2}$ **(B)** 3 **(C)** $\frac{7}{2}$ **(D)** 5 **(E)** 8

23. A 4 by 4 "antimagic" square is an arrangement of the numbers 1 to 16 inclusive in a square, so that the totals of each of the four rows and four columns and two main diagonals are ten consecutive numbers in some order. The diagram shows an incomplete antimagic square. When it is completed, what number will replace the squiggly Greek letter?

4	5	7	14
6	13	3	ξ
11	12	9	
10			

(A) 1 **(B)** 2 **(C)** 8 **(D)** 15 **(E)** 16

24. A tennis club has n left-handed players and $2n$ right-handed players, but in total there are fewer than 20 players. (Neither Ruth Malinowski nor any other ambidexterous tennis pro competed.) At last summer's tournament, in which every player in the club played every other player exactly once, no matches were tied and the ratio of the number of matches won by left-handed players to the number of matches won by right-handed players was $3 : 4$. The value of n is

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** not determined by this information

25. Let $f(n)$ be the number of ways to write n as a sum of powers of 2, where each power of 2 is used 0, 1, or 2 times. For example, $4 = 2^2 = 2^1 + 2^1 = 2^0 + 2^0 + 2^1$, so $f(4) = 3$. The value of $f(130)$ is

(A) 11 **(B)** 12 **(C)** 13 **(D)** 15 **(E)** 16