The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
and
The Canadian Mathematics Competition
presents

The Third Annual Small c Competition

for First and Second Year Students

Saturday 04 October 2003

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open the contest booklet until you are told to do so.
2. You may use slide rules, abaci, rulers, protractors, compasses and paper for rough work. You may also use log tables; log cabins are not permitted.
3. On your response form, print your name, program (plan?), and ID number.
4. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, enter it in the appropriate box on the response form.
5. Your response form will be read only by a *dumb human*, who has undergone rigorous training in order to be able to recognize the letters **A** through **E**. For your own sake, please write neatly.
6. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
7. Diagrams are *not* drawn to scale. They are intended as aids only.
8. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A

1. The first derivative of x^2, with respect to x, is

   (A) x **(B)** x^2 **(C)** 2 **(D)** $2x$ **(E)** $\frac{1}{3}x^3$

2. Peter pays Nerissa \$19 per hour, but decides to cut Ian's pay to \$11 per hour. If Nerissa and Ian each work 40 hours per week for 8 eight weeks, how much more does Nerissa make than Ian over this period?

   (A) \$2560 **(B)** \$3520 **(C)** \$6080 **(D)** \$3200 **(E)** \$2400

3. The two-digit perfect square whose digits have a product which is also a perfect square is

   (A) 64 **(B)** 66 **(C)** 49 **(D)** 36 **(E)** 81

4. Yesterday, the reading on Carolyn's electricity meter was 098657 units. She was shocked to realize that all six of these digits are different. How many more units of electricity will she use before the next time all the digits are different?

   (A) 1 **(B)** 4 **(C)** 14 **(D)** 55 **(E)** 3688

5. If $\dfrac{x-3y}{y} = 12$, then $\dfrac{x}{y}$ is equal to

   (A) 9 **(B)** 2 **(C)** 10 **(D)** 3 **(E)** 15

6. There were 5 parrots in a pet store. The average price of the 5 parrots was \$600. One day, the one parrot that was not dead or sleeping was sold. The average price of the remaining 4 birds was \$500. What was the price of the parrot that was sold?

   (A) \$100 **(B)** \$200 **(C)** \$550 **(D)** \$600 **(E)** \$1000

7. If $1 - \dfrac{4}{x} + \dfrac{4}{x^2} = 0$, then $\dfrac{2}{x}$ is equal to

   (A) -1 **(B)** 1 **(C)** 2 **(D)** -2 **(E)** 4

8. In the diagram, a rectangular prism was made out of three blocks, each consisting of four cubes. Which of the blocks below has the same shape as the white block?

   (A) **(B)** **(C)**

   (D) **(E)**

9. Which of the numbers below is odd regardless of the value of the natural number n?

(A) $2003n$ **(B)** $n^2 + 2003$ **(C)** n^3 **(D)** $n + 2004$ **(E)** $2n^2 + 2003$

10. Erin's Combinatorics prof gives her 20 problems to prepare for the exam. The exam will have 8 problems on it of which she must answer 5. What is the least number of problems that she must prepare to be guaranteed that she does not have to answer a problem that she has not prepared?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

Part B

11. Maddie has two strange dice. The faces of each die show the numbers 1 to 6 as usual, but the odd numbers are negative (ie. -1, -3 and -5 in place of 1, 3 and 5). If she throws the two dice, which total cannot be achieved?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 8

12. In the diagram, the value of y is

(A) 71 **(B)** 59 **(C)** 66 **(D)** 76 **(E)** 94

13. It was reported recently that, in an average lifetime of 70 years, each human is likely to swallow about 8 spiders while sleeping. If the population of Canada is around 30 million, which of the following is the best estimate of the number of unfortunate spiders consumed this way in Canada each year?

(A) 30 000 **(B)** 300 000 **(C)** 3 000 000 **(D)** 30 000 000 **(E)** 300 000 000

14. A box contains 1 gold ball and 2 black balls. June, April and May in order each select a ball at random from the box and do not replace the ball they select. If the woman who chooses the gold ball wins, what is the probability that April wins?

(A) 1 **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{\text{July}}{\text{September}}$

15. Let $f(x)$ be a function such that for each integer x,

$$f(x) = x^2 f(x-1)$$

If $f(1) = 10$, then $f(3)$ is equal to

(A) 60 **(B)** 120 **(C)** 180 **(D)** 10 **(E)** 360

16. Two identical trucks each contain n large containers. These trucks are used by the Bookstore to transport 505 calculus textbooks to the dump. In the first truck, Judith loads 21 textbooks into each of the n large containers. On the second truck, Judith notices that if she loads 20 textbooks into each of the n containers, then not all 505 textbooks can be sent. She also determines that if she tries to load 23 textbooks into each of the n containers on the second truck, she will run out of books before all of the containers are full, leaving at least 1 container empty. How many containers does each truck hold?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

17. The number $a = 111\ldots11$ is made out of 2003 digits, all equal to 1. What is the sum of the digits of the number $2003 \times a$?

(A) 10 000 **(B)** 10 015 **(C)** 10 020 **(D)** 10 030 **(E)** 2003×2003

18. The positive value of x which minimizes

$$(x^2-4)^2-4(x^2-4)+18$$

is closest to

(A) 2.25 **(B)** 2.35 **(C)** 2.45 **(D)** 2.55 **(E)** 2.65

19. Define two sequences a_n and b_n by $a_n = 4n^2+625$ and $b_n = 100n+16$, for $n = 1, 2, 3, \ldots$. The smallest value of $n \geq 1$ such that $a_n < b_n$ is

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

20. The width : height ratio of television screens is changing from the traditional 4 : 3 to the widescreen 16 : 9. If a traditional screen and a widescreen have the same area, then what is the ratio widescreen width : traditional width?

(A) $2 : \sqrt{3}$ **(B)** 3 : 2 **(C)** $3 : \sqrt{2}$ **(D)** 4 : 3 **(E)** $4 : \sqrt{6}$

Part C

21. The letters S, M, A, L, and L are written on five tiles, one letter per tile, and then placed in a black box. If Sarah chooses two of the tiles at random, what is the probability that she chooses both L's?

(A) $\frac{1}{15}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{9}$ **(E)** $\frac{1}{8}$

22. In the multiplication $A6BC \times 7 = D9E98$, each letter represents a digit. Which letter represents 3?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

23. How many natural numbers n have the property that out of all of the positive divisors of n, which are different from 1 and n, the greatest one is 15 times greater than the smallest one?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 0 **(E)** Infinitely many

24. The numbers $4, 6, 8, \ldots, 28$ form an arithmetic sequence. A number n is the sum of five distinct numbers from this sequence. How many values of n are there?

(A) 39 **(B)** 41 **(C)** 43 **(D)** 45 **(E)** 47

25. If a, b and c are the roots of the equation $3x^3-7x-12=0$, an equation whose roots are $\dfrac{1}{a-2}$, $\dfrac{1}{b-2}$, and $\dfrac{1}{c-2}$ is

(A) $12u^3+65u^2+116u+65=0$
(B) $2u^3-11u^2-18u-12=0$
(C) $2u^3-29u^2-18u-3=0$
(D) $22u^3-29u^2+18u-3=0$
(E) $3u^3+18u^2+29u+2=0$