

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
and
The Canadian Mathematics Competition
presents*

The Fourth Annual Small c Competition

for First and Second Year Students

Saturday 02 October 2004

Time: 1 hour

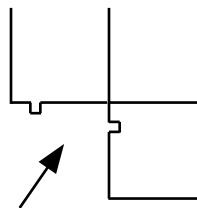
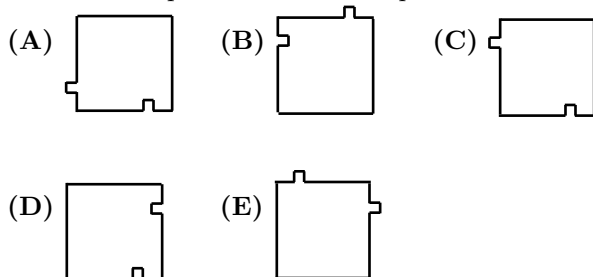
Calculators are permitted.

Instructions:

1. Do not open the contest booklet until you are told to do so.
2. You may use slide rules, abaci, rulers, protractors, compasses and paper for rough work. You may also use log tables; log cabins are not permitted.
3. On your response form, print your name, plan, and ID number.
4. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, enter it in the appropriate box on the response form.
5. Your response form will be read only by a *dumb human*, who has undergone rigorous training in order to be able to recognize the letters **A** through **E**. For your own sake, please write neatly.
6. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
7. Diagrams are *not* drawn to scale. They are intended as aids only.
8. Als u dit kunt lezen, spreekt u het Nederlands.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A

1. Which of the pieces fits into the space shown?

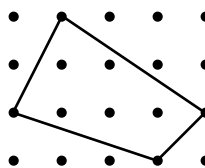


2. The sum of the series $2 + 5 + 8 + 11 + \dots + 50$ is

(A) 90 (B) 425 (C) 416 (D) 442 (E) 495

3. Pegs are put in a board 1 cm apart both horizontally and vertically. A rubber band is stretched over four pegs, as shown, forming a quadrilateral. Its area, in square centimetres, is

(A) 4 (B) 4.5 (C) 5
(D) 5.5 (E) 6



4. The Detroit Bengals have won 25 games out of 75 played and still have 45 games to play. How many of the remaining games must they win so that the percentage of games won for the entire season is 30%?

(A) 11 (B) 36 (C) 13.5 (D) 45 (E) 25

5. While putting up the Pink Tie, Judith single-handedly puts a 25 m long ladder against a vertical wall of the Math Building. The foot of the ladder is 7 m from the base of the building. An Engineer starts to steal the ladder and moves the top of the ladder 4 m down the wall. How much farther, in metres, does the Engineer move the foot of the ladder *away* from the wall?

(A) 9 (B) 15 (C) 5 (D) 8 (E) 4

6. In a certain year, January had exactly four Tuesdays and four Saturdays. On what day did January 1 fall that year?

(A) Monday (B) Tuesday (C) Wednesday (D) Friday (E) Saturday

7. The 11 forwards of the Waterloo Worriers scored 440 goals in total. If seven of these players averaged 52 goals each, then the average number of goals scored by the remaining four players was

(A) 19 (B) 28 (C) 36 (D) 40 (E) 76

8. The points $(1, a)$ and $(-1, b)$ lie on the curve $y = px^2 + qx + 5$. If $a + b = 14$, then the value of p is

(A) 2 (B) 7 (C) 5 (D) 8 (E) 14

9. Two cyclists are 168 km apart and set out towards each other at 8 a.m. They meet at 11 a.m. If one travels at 30 km/h, how fast does the other travel?

(A) 30 km/h (B) 138 km/h (C) 56 km/h (D) 26 km/h (E) 56 km/h

10. The positive integers are written in rows

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
⋮	⋮	⋮	⋮	⋮

The row which has a sum nearest in value to 2004 is row number

- (A) 4 (B) 80 (C) 81 (D) 82 (E) 2005

Part B

11. If we define $a\nabla b = a^3 + b^3$, then a value of x that makes $1\nabla 12 = x\nabla 9$ is

- (A) 4 (B) 11 (C) 10 (D) 9 (E) 8

12. Any six-digit number formed by repeating a three-digit number (for example, 265 265 or 345 345) is divisible by

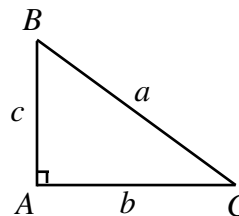
- (A) 7 only (B) 5 only (C) 13 only (D) 101 (E) 1001

13. The lowest common multiple of two positive integers is 378 and their greatest common divisor is 6. One of the two integers is 42. What is the other integer?

- (A) 9 (B) 54 (C) 63 (D) 126 (E) 336

14. In $\triangle ABC$, $\cos A + \sin B + \tan C$ equals

- (A) $\frac{b^2 + ac}{ab}$ (B) $\frac{b + c}{a + b}$ (C) $\frac{a + b + c}{b}$
 (D) $\frac{2c}{a + b}$ (E) $\frac{ab + b^2 + ac}{ac}$



15. The point of intersection of the lines $x - 3y - 8 = 0$ and $2x - y - 6 = 0$ also lies on the line $kx + 5y - 12 = 0$. The value of k is

- (A) 2 (B) 11 (C) -5 (D) 1 (E) 8

16. The roots of the equation $x^3 + \left(a - \frac{1}{a}\right)x^2 - x = 0$ are

- (A) 0, a , 1 (B) a , $-\frac{1}{a}$ (C) $-a$, $\frac{1}{a}$ (D) 0, a , $-\frac{1}{a}$ (E) 0, $-a$, $\frac{1}{a}$

17. If $10^{2005} - 25$ is expressed as an integer, the sum of its digits is

- (A) 18 027 (B) 18 039 (C) 18 015 (D) 18 034 (E) 18 048

18. If $f(x + 1) = x^2 + 5x + 3$, then $f(x - 1)$ is equal to

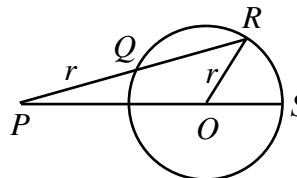
- (A) $x^2 + 5x + 1$ (B) $x^2 + x - 3$ (C) $x^2 - 5x + 1$ (D) $x^2 + x + 3$ (E) $x^2 - 5x - 1$

19. A , B , C and D are four different digits, and are used to form three-digit numbers BAD , CAD and DAB . What is the largest possible value of $BAD + CAD + DAB$?

- (A) 2906 (B) 2606 (C) 2597 (D) 2536 (E) 2726

20. In the diagram, POS is a straight line through the centre O of a circle of radius r . The line PQR is drawn so that $PQ = r$. If $\angle ROS = 60^\circ$, then $\angle OPQ$ equals

- (A) 10° (B) 15° (C) 20°
 (D) 25° (E) 30°



Part C

21. If Serge and Ross work together, it would take them 2 hours to put Pink Tie stickers on a roomful of calculators, while Ross and Lis would take 3 hours, and Lis and Serge would take 4 hours to complete this same job. How long, in hours, would all three take if they worked together?

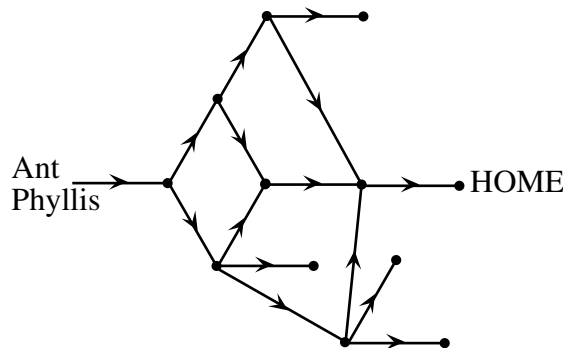
(A) $\frac{5}{3}$ (B) $\frac{5}{2}$ (C) 1 (D) $\frac{24}{13}$ (E) $\frac{8}{5}$

22. The value of the constant k so that the function defined by $f(x) = \frac{x+5}{x+k}$ will be its own inverse is

(A) 1 (B) 5 (C) $-\frac{1}{2}$ (D) 3 (E) -1

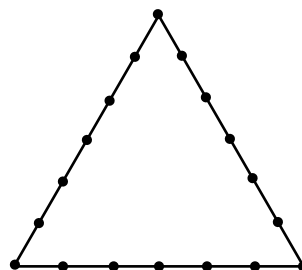
23. Ant Phyllis is travelling home on the network of tracks. She cannot turn back. When she reaches a junction, there is an equal probability of her choosing any forward path. The probability that she will reach home is

(A) $\frac{1}{2}$ (B) $\frac{11}{18}$ (C) $\frac{29}{72}$
 (D) $\frac{23}{36}$ (E) $\frac{43}{72}$



24. In the diagram, how many triangles can be formed using three of the 18 points as vertices?

(A) 816 (B) 777 (C) 717
 (D) 711 (E) 811



25. A tetrahedron has vertices $O(0, 0, 0)$, $A(1, 0, 0)$, $B(2, 3, 0)$ and $C(0, 0, 4)$. The shortest distance between the edges OA and BC is

(A) 3 (B) $\frac{12}{5}$ (C) $\frac{14}{5}$ (D) 2 (E) $\frac{13}{5}$