The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
and
The Canadian Mathematics Competition
presents

The Fifth Annual Small c Competition

for First and Second Year Students

Saturday 01 October 2005

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open the contest booklet until you are told to do so.
2. You may use slide rules, abaci, rulers, protractors, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. This year, Tom-toms and Coleman stoves are also permitted for the first time.
3. On your response form, print your name, plan, and ID number.
4. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, enter it in the appropriate box on the response form.
5. Your response form will be read only by a *dumb human*, who has undergone rigorous training in order to be able to recognize the letters **A** through **E**. For your own sake, please write neatly.
6. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 questions.
7. Diagrams are *not* drawn to scale. They are intended as aids only.
8. Als u dit kunt lezen, spreekt u het Nederlands.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Part A

1. The value of $(1-2)-(3-4)-(5-6)-(7-8)$ is

   (A) 2 **(B)** -2 **(C)** -1 **(D)** 1 **(E)** 0

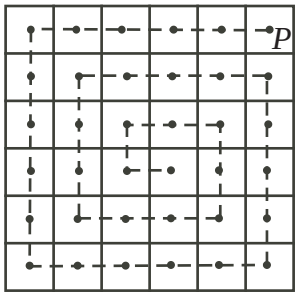
2. Fiona has 48 Toyota Corollaries. Half of them are blue, one quarter are red, and one sixth are green. How many cars of other colours does she have?

   (A) 6 **(B)** 4 **(C)** 8 **(D)** 2 **(E)** 12

3. On March 14 at 1:59 p.m., eighteen students participated in a pi-reciting contest. The students were divided into pairs numbered from 1 to 9. Each pair with an even number consisted of an Accounting student and a Business student, and each pair with an odd number consisted of two Business students. How many Business students participated in the contest?

   (A) 10 **(B)** 12 **(C)** 14 **(D)** 11 **(E)** 18

4. In the diagram, a crumb is placed at the centre of each small square of a 6×6 grid, where each small square is 1 cm by 1 cm. Ant Phyllis starts at P in the upper right corner of the grid and eats the crumb at P. She then walks to the left, then down, then to the right, then up, etc. as shown. What is the total distance that she walks in order to eat all of the crumbs?

   (A) 36 **(B)** 35 **(C)** 24
   (D) 34 **(E)** 32

5. Of the following points, which is *not* situated on the line passing through $P(1,-3)$ and $Q(-2,6)$?

   (A) $(0,0)$ **(B)** $(-1,-3)$ **(C)** $(2,-6)$ **(D)** $(-1,3)$ **(E)** $(3,-9)$

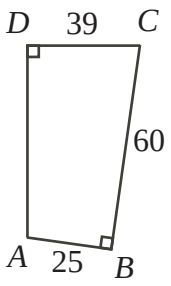
6. During Frosh Week, 3 Mathies and 2 Engineers weigh 368 kg while 7 Mathies and 4 Engineers weigh 792 kg. How much do 2 Mathies and 1 Engineer weigh?

   (A) 156 kg **(B)** 256 kg **(C)** 232 kg **(D)** 212 kg **(E)** 192 kg

7. If $x^2 - x - 2 = 0$, the value of $1 - \dfrac{1}{x} - \dfrac{2}{x^2}$ is

   (A) 0 **(B)** -1 **(C)** 2 **(D)** -2 **(E)** 1

8. In the diagram, $\angle ABC = \angle CDA = 90°$. Also, $AB = 25$, $BC = 60$ and $CD = 39$. What is the area of quadrilateral $ABCD$?

   (A) 1776 **(B)** 1867 **(C)** 2514
   (D) 1820 **(E)** 1764

9. The vertices of a cube are coloured black or gold. The centre point of each face is also coloured according to the following rule:

> If the face has an odd number of gold vertices, the centre of the face is coloured green, otherwise it is coloured red.

If exactly two of cube's vertices are coloured black, what is the maximum number of faces that could have a red point at their centre?

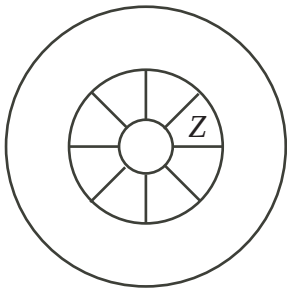
(A) 6 **(B)** 4 **(C)** 3 **(D)** 2 **(E)** 1

10. How many three-digit numbers are there in which all the digits are even?

(A) 25 **(B)** 64 **(C)** 75 **(D)** 100 **(E)** 125

Part B

11. In the diagram, a sky-diving target is shown, constructed from circles of radii 1 km, 3 km and 6 km. The middle ring is divided into eight identical zones. When Barry jumps from the helicopter, he is equally likely to hit any point inside the largest circle. What is the probability that he will hit zone Z?



(A) $\frac{1}{36}$ **(B)** $\frac{1}{32}$ **(C)** $\frac{1}{18}$
(D) $\frac{1}{16}$ **(E)** $\frac{1}{64}$

12. The number of points of intersection between the two parabolae $y = -2x^2$ and $y^2 = 8\sqrt{2}x$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13. The number of angles θ with $0 \leq \theta < 2\pi$ which are solutions to $\cos(\theta)\sin(2\theta)\cos(3\theta) = 0$ is

(A) 8 **(B)** 10 **(C)** 12 **(D)** 6 **(E)** 4

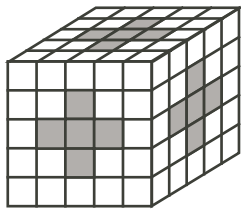
14. In a room in Mackenzie King Village, two bottles of the same volume were filled with a mixture of water and "juice". The ratios of water to juice in these bottles are 2 : 1 and 4 : 1 respectively. If we pour the mixture from both bottles into one container, the ratio of the volume of water to the volume of juice will be equal to

(A) 3 : 1 **(B)** 6 : 1 **(C)** 8 : 1 **(D)** 15 : 4 **(E)** 11 : 4

15. The new "fine print" version of Stewart's *Calculus* has 192 pages. If each page from 1 to 192 is numbered, how many digits are used in total for the numbering?

(A) 466 **(B)** 467 **(C)** 464 **(D)** 465 **(E)** 468

16. A $5 \times 5 \times 5$ cube is formed using $1 \times 1 \times 1$ cubes. Some of the small cubes are removed by punching out cubes all the way from front to back, from top to bottom and from side to side in the darkened regions shown. How many of the small cubes are left?



(A) 76 **(B)** 108 **(C)** 68
(D) 60 **(E)** 88

17. The sum of 6 consecutive whole numbers cannot equal

(A) 33 **(B)** 303 **(C)** 333 **(D)** 330 **(E)** 3303

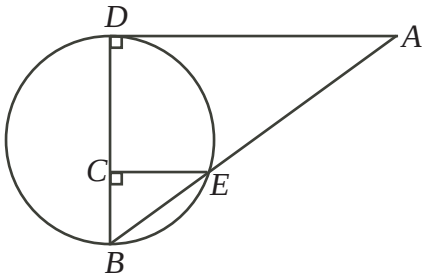
18. The function $f(x)$ is defined as

$$f(x) = \begin{cases} 2^{-x} - 1 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$

If $f(x) > 1$, then the possible values for x are

(A) $-1 < x < 1$ **(B)** $-1 < x$ **(C)** $x < 0$ or $x > 1$
(D) $x < -1$ or $x > 1$ **(E)** $x > 1$

19. In the diagram, $BC = 2$, $CD = 4$ and AD is tangent to the circle at D. What is the length of AE?



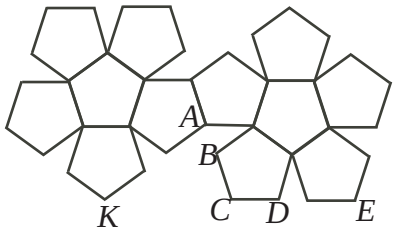
(A) 6 **(B)** $6\sqrt{3}$ **(C)** $5\sqrt{2}$
(D) 10 **(E)** $4\sqrt{3}$

20. The notation $n!$ denotes the product $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. For example, $4! = 4 \times 3 \times 2 \times 1 = 24$. If n^3 is a divisor of 12!, what is the largest possible positive integer value for n?

(A) 24 **(B)** 12 **(C)** 8 **(D)** 72 **(E)** 40

Part C

21. In the diagram, a net of a dodecahedron is shown. When the net is folded to make a dodecahedron, which vertex will meet K?



(A) A **(B)** B **(C)** C
(D) D **(E)** E

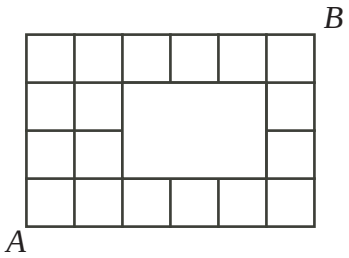
22. If $g(x) = \max\{-2x^2 - 4x + 19, -x^2 + 6x + 8\}$ for $-2 \leq x \leq 5$ (that is, for any value of x in the domain, $g(x)$ is equal to whichever of $-2x^2 - 4x + 19$ and $-x^2 + 6x + 8$ is larger), what is the maximum value of $g(x)$?

(A) 8 **(B)** 19 **(C)** 13 **(D)** 17 **(E)** 21

23. The number 24 has 8 positive divisors (1, 2, 3, 4, 6, 8, 12, 24) while the number 25 has 3 positive divisors (1, 5, 25). If a, b and c are consecutive positive integers that each have n positive divisors, what is the smallest possible value of n?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

24. In the road-map shown in the diagram, each line segment represents a street which can only be travelled along in either the rightwards or upwards direction. How many paths are there from point A to point B?



(A) 70 **(B)** 72 **(C)** 84
(D) 80 **(E)** 75

25. If x, y and z are real numbers such that

$$x^2 + y^2 + z^2 - xy - yz - zx = 12$$

then the maximum possible difference between any two of x, y and z is

(A) $2\sqrt{2}$ **(B)** 4 **(C)** 3 **(D)** $2\sqrt{6}$ **(E)** $2\sqrt{3}$