The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
and
The Canadian Mathematics Competition
presents

The Sixth Annual Small c Competition

for First and Second Year Students

Saturday 00 October 2006

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open the Contest booklet until you are told to do so.

10. You may use slide rules, abaci, rulers, protractors, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. This year, Tom-toms and Coleman stoves are again permitted.

11. On your response form, print your name, plan, and ID number.

100. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, enter it in the appropriate box on the response form.

101. Your response form will be read only by a *dumb human*, who has undergone rigorous training in order to be able to recognize the letters **A** through **E**. For your own sake, please write neatly.

110. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.

111. Diagrams are *not* drawn to scale. They are intended as aids only.

1000. Als u dit kunt lezen, spreekt u het Nederlands.

1001. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

1010. In typical Canadian fashion, "EH" is the answer many times. Or maybe it isn't...

Part A

1. The value of $\dfrac{1}{\frac{1}{2}+\frac{1}{2}}+\frac{1}{2}$ is

   (A) $\frac{3}{2}$ **(B)** $\frac{9}{2}$ **(C)** $\frac{3}{4}$ **(D)** 1 **(E)** 2

2. Last Sunday morning, the numbers of men and women frittering the time away at the Church of St. Timothy of Horton were in the ratio 5 : 8. If there were 91 in total in attendance, the number of men was

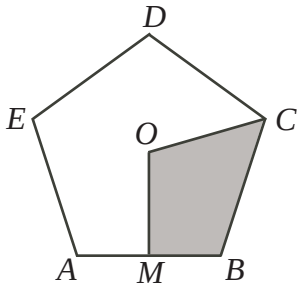
   (A) 35 **(B)** 30 **(C)** 56.875 **(D)** 25 **(E)** 40

3. The expression $7(x-y)-2(3x-4y)$ equals

   (A) $x-15y$ **(B)** $x+y$ **(C)** $x-3y$ **(D)** $x-5y$ **(E)** $x+7y$

4. In the diagram, O is the centre of a regular pentagon $ABCDE$ and M is the midpoint of AB. What percentage of the area of the whole pentagon is shaded?

   (A) 10% **(B)** 20% **(C)** 30%
   (D) 40% **(E)** 50%



5. Which of the following numbers is not the difference between two of the others?

   (A) 1 **(B)** 7 **(C)** 3 **(D)** 5 **(E)** 2

6. Maddie's average on her first 8 assignments in MATH 135 is 78%. What mark must she get on Assignment #9 to receive an average of 80% over the 9 assignments?

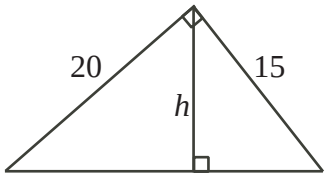
   (A) 108% **(B)** 80% **(C)** 84% **(D)** 96% **(E)** 88%

7. In the sequence 1, 3, 7, 15, 31, ..., each term is 1 more than double the previous term.
   The 10th term is

   (A) 513 **(B)** 1007 **(C)** 979 **(D)** 511 **(E)** 1023

8. In the diagram, the length of h is

   (A) 12 **(B)** $10\sqrt{3}$ **(C)** $8\sqrt{2}$
   (D) 10 **(E)** $5\sqrt{5}$



9. Serge has 3 hours to ride 60 km. He pedals along at 20 km/h for 90 minutes, but then has to wait 40 minutes for a bridal train to pass by. In his ensuing serge (sic) of power, how fast must he pedal to complete the distance in 3 hours?

   (A) 36 km/h **(B)** 20 km/h **(C)** 45 km/h **(D)** 25 km/h **(E)** 30 km/h

10. If x, y and z are all different negative integers, and $A=3x+2y+z+xyz$, then the largest possible value of A is

    (A) -20 **(B)** -17 **(C)** -16 **(D)** -10 **(E)** -9

Part B

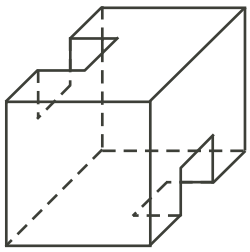
11. Bev, Ian and Prabhakar decide to put their money together to buy a ginormous pink tie. Bev has 60% of the money needed and Ian has 40% of the remaining amount. After Bev and Ian pool their money together, they ask Prabhakar for his \$30, which gives them exactly enough money to buy the tie. How much does the tie cost? (Note: Ginormous pink ties are not subject to PST or GST.)

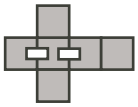
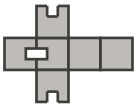
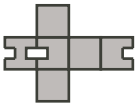
(A) \$50 **(B)** \$60 **(C)** \$125 **(D)** \$150 **(E)** \$200

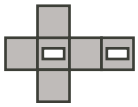
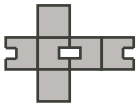
12. The number of pairs of integers (a, b) with $0 \leq a < b \leq 9$ such that $10a + b$ and $10b + a$ are both prime numbers is

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

13. While packing up his books to move from his office, the latest "Has Dean" in the Faculty of Math used only boxes similar to the one in the diagram. Which of the following shapes did Alan have to fold to make these boxes?



(A)  **(B)**  **(C)** 

(D)  **(E)** 

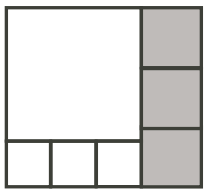
14. If x and y are positive integers, then the largest value of $x + y$ such that $2x + 3y = 228$ is

(A) 76 **(B)** 77 **(C)** 115 **(D)** 114 **(E)** 113

15. With how many zeroes does the product of the first 2006 prime numbers end?

(A) 0 **(B)** 500 **(C)** 3 **(D)** 427 **(E)** 1

16. At UW's new MaRS office, its rectangular office area is divided into 7 smaller *square* offices. The length of each side of each of the shady offices is 8 m. What is the length of a side of the big white square office?



(A) 16 m **(B)** 20 m **(C)** 30 m
(D) 24 m **(E)** 18 m

17. The positive integers x and y differ by 2. Their squares differ by 2008. The sum of x and y is

(A) 2008 **(B)** 1006 **(C)** 1002 **(D)** 2006 **(E)** 1004

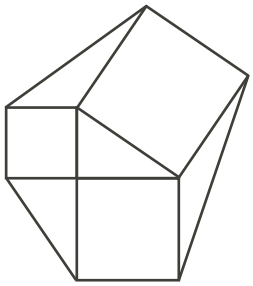
18. While working on a problem from an old Small c Contest, Lino noticed that the following conclusions are correct:

- If answer A is correct, then answer B is also correct.
- If answer E is not correct, then answer B is also not correct.
- If answer B is not correct, then neither D nor C is correct.

Lino, who is smarter than the average grad student, knows that only one answer can be correct. Please help Lino! Which answer is correct?

(A) D **(B)** E **(C)** A **(D)** C **(E)** B

19. In the diagram, squares are drawn externally on the sides of a 3-4-5 triangle. When the tips of the squares are joined, a funny-looking hexagon is formed. The area of this hexagon is



(A) 50 **(B)** 86 **(C)** 98
(D) 96 **(E)** 74

20. For all real values of x for which the terms are defined

$$\cot\left(\tfrac{1}{4}x\right) - \cot x = \frac{\sin(kx)}{\left(\sin\left(\tfrac{1}{4}x\right)\right)(\sin x)}$$

The value of k is

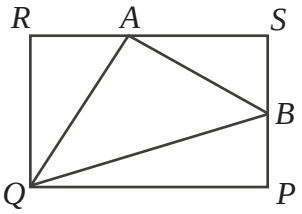
(A) $\frac{3}{8}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{4}$ **(E)** $\frac{3}{2}$

Part C

21. If $2^a = 3$, $3^b = 4$, $4^c = 5$, $5^d = 6$, $6^e = 7$, and $7^f = 8$, then $abcdef$ equals

(A) $\dfrac{\ln 33}{\ln 27}$ **(B)** $\dfrac{\ln 20160}{\ln 5040}$ **(C)** $\dfrac{\ln 8}{\ln 3}$ **(D)** 4 **(E)** 3

22. In the diagram, rectangle $PQRS$ has $PQ = 5$ and $QR = 2$. Also, A is a point on RS and B is a point on PS such that the area of $\triangle QAB$ is 4. The smallest possible value of $PB + AR$ is closest to

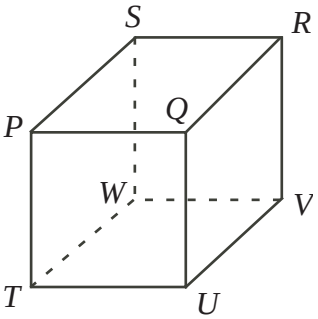


(A) 2.6 **(B)** 2.7 **(C)** 2.8
(D) 2.9 **(E)** 3.0

23. In a dice game for winner-take-all Math/Engineering supremacy, Adel rolls one die and Tom rolls 2 dice. If the number that Adel rolls is greater than the largest number that Tom rolls, Adel wins. Otherwise, Tom wins. What is the probability that Tom wins?

(A) $\frac{7}{12}$ **(B)** $\frac{161}{216}$ **(C)** $\frac{125}{216}$ **(D)** $\frac{15}{24}$ **(E)** $\frac{41}{72}$

24. Given the cube $PQRSTUVW$ as shown, the plane which passes through P and the centres of faces $TUVW$ and $UQRV$ intersects UV at X. The ratio $UX : XV$ is



(A) 2 : 1 **(B)** 3 : 2 **(C)** 3 : 1
(D) 5 : 4 **(E)** 5 : 2

25. A total of 2006 circles, each circle intersecting each other in two distinct points, are in a plane and no three of them pass through the same point. They divide the plane into a number of disjoint regions which, including the exterior of all circles, is

(A) 4 022 072 **(B)** 4 022 012 **(C)** 4 022 092 **(D)** 4 022 032 **(E)** 4 022 052