

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents*

The Seventh Annual Small c Competition

for First and Second Year Students

Saturday 22 September 2007

Time: 1 hour

Calculators are permitted.

Instructions:

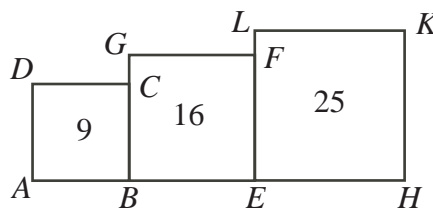
1. Do not open the Contest booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, protractors, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. This year, Tom-toms and Coleman stoves are again permitted.
11. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
100. On your response form, print your name, plan, and ID number.
101. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
110. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.
111. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
1000. Diagrams are *not* drawn to scale. They are intended as aids only.
1001. Als u dit kunt lezen, spreekt u het Nederlands.
1010. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.
1011. In typical Canadian fashion, “EH” is the answer many times. Or maybe it isn’t...

Part A

- The sum $2007 + 207 + 27$ equals
 (A) 22277 (B) 999 (C) 2231 (D) 2241 (E) π
- Clara likes Pink Tie stickers. There are 12 stickers in strip and 16 strips of stickers per package. If Clara has 1152 stickers (just enough for every calculator), how many packages of stickers does she have?
 (A) 6 (B) 12 (C) 69 (D) 8 (E) 1124
- If $x + 3y + 4z = 14$ and $x = 2 - 3y$, then z equals
 (A) 3 (B) 12 (C) 4 (D) 3.5 (E) 2
- Let $S(n)$ denote the sum of the digits of the integer n (written as usual in base 10). For example, $S(64) = 10$. What is the value of $(13 - S(13)) - (12 - S(12))$?
 (A) 0 (B) 1 (C) -1 (D) 11 (E) 3

- In the diagram, $ABCD$, $BEFG$ and $EHLK$ are squares with areas 9, 16 and 25, respectively. $ABEH$ is a straight line. The length of AK is

- (A) 13 (B) 12 (C) $2\sqrt{50}$
 (D) 15 (E) $3\sqrt{17}$

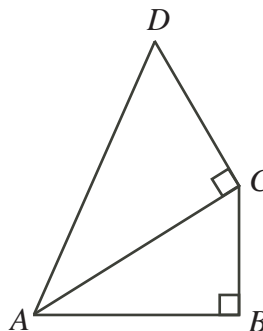


- A parabola passes through $(0, 0)$ and has its vertex at $(-2, -3)$. One of its x -intercepts is
 (A) -2 (B) -6 (C) -4 (D) -1 (E) -3
- Hillary and Bill go fishing.
 Hillary catches 8 fish and tells us that the average weight of the fish is 12 pounds.
 Bill catches 10 fish and tells us that the average weight of the fish is 19.2 pounds.
 What is the average of the weights of all of the fish that they caught?
 (Of course, stories about fish are always true.)
 (A) 15.6 pounds – honest!
 (B) 16 pounds – really!
 (C) 14.4 pounds – trust me!
 (D) 9 pounds – I cannot tell a lie.
 (E) 14.6 pounds – you should have seen the one that got away...

- How many perfect squares appear in the set $\{99, 100, 101, \dots, 9999, 10000, 10001\}$?
 (A) 89 (B) 91 (C) 891 (D) 100 (E) 216

- In the diagram, $AB = 12$, $\angle ABC = \angle ACD = 90^\circ$ and $\angle CAB = \angle DAC = 30^\circ$. What is the length of AD ?

- (A) $10\sqrt{3}$ (B) 24 (C) 9
 (D) 6 (E) 16



10. In honour of Canada's 31st Governor General and a wacky math teacher from Windsor, the cells of a 3×3 grid are coloured Grey and White as shown in Figure 1. One move allows us to exchange any two cells positioned in the same row or in the same column. What is the least number of moves necessary to obtain Figure 2?

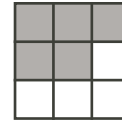


Figure 1

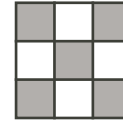


Figure 2

- (A) 1 (B) 2 (C) 3
(D) 4 (E) 5

Part B

11. A very dense die with dimensions $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$ has a mass of 54 kg. A second die made of the same extremely dense material as the first (that is, made of recycled copies of Stewart's *Calculus*) has dimensions $4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}$. The mass of the second die is

- (A) 108 kg (B) 96 kg (C) 72 kg (D) 128 kg (E) 216 kg

12. In the magic square, each row, column and diagonal has the same sum. Which of the five letters has the largest value?

-3	a	22
b	2	c
d	e	7

- (A) c (B) d (C) a
(D) e (E) b

13. The line $y = 3x + b$ passes through the points $(-1, 2)$ and $(k, 11)$. The value of k is

- (A) -2 (B) $\frac{11}{3}$ (C) 4 (D) 2 (E) 5

14. The sum of all of the positive integers from 250 to 750 (including both 250 and 750) is

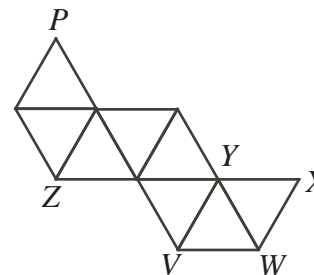
- (A) 500 000 (B) 501 000 (C) 250 500 (D) 250 250 (E) 250 000

15. Jillian and Janelle each choose an integer (possibly the same one) between 1 and 5. The probability that the sum of their integers is even is

- (A) $\frac{1}{2}$ (B) $\frac{13}{25}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$ (E) $\frac{9}{25}$

16. In the diagram, the figure (called a *net*) can be folded to make an octahedron. When it is folded, the point labelled P coincides with the point labelled

- (A) V (B) W (C) X
(D) Y (E) Z



17. Bev jumps out of an airplane that is flying at 1230 m, wearing a parachute that keeps her falling at a constant 10 m/s. At the same instant, Serge jumps out of a second plane flying at a height of 1500 m, but his parachute controls his descent at 15 m/s. (Serge and Bev had previously synchronized their watches to a clock in the Math Building, which was, of course, completely wrong.) What height above the ground will Bev and Serge reach at the same time?

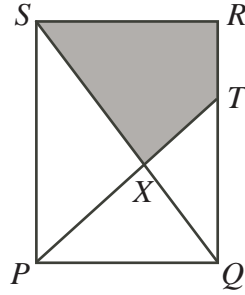
- (A) 690 m (B) 123 m (C) 540 m (D) 810 m (E) 0 m

18. p , q and r are prime numbers. If $p + q + r = 229$ and $p - q - r = 217$, then pqr equals

- (A) 1784 (B) 1338 (C) 2230 (D) 669 (E) 2007

19. In the diagram, $PQRS$ is a rectangle with T on QR . PT and SQ intersect at X . If the area of $\triangle PXS$ is 8 and the area of $\triangle PXQ$ is 4, what is the shaded area?

- (A) 8 (B) 6 (C) 12
(D) 10 (E) 9



20. Every polyhedron with V vertices, E edges and F faces satisfies Euler's Formula, $V - E + F = 2$. (As far as we know, Euler never visited Edmonton.) An Elongated Pentagonal Orthocupolarotunda is a polyhedron with exactly 37 faces, 15 of which are squares, 7 of which are regular pentagons, and 15 of which are triangles. How many vertices does it have?

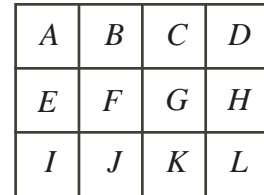
- (A) 37 (B) 45 (C) 35 (D) 42 (E) 40

Part C

21. Last year, in the Choir of St. Thomas of Colemania, there were 30 more men than women. This year, the total number of choir members has increased by 10%, while the number of women has increased by 20% and the number of men has increased by 5%. How many members are in the Choir this year?

- (A) 88 (B) 99 (C) 110 (D) 121 (E) 132

22. Ron, who incidentally was last year's Ontario Tetris champion, has a 3 by 4 sheet of twelve stamps. He wishes to select four stamps, attached to each other by at least one edge. He could select $ABCD$ or $ABFG$ but not $IJGL$. How many different combinations of four stamps are possible?



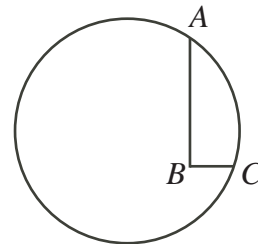
- (A) 68 (B) 62 (C) 61
(D) 63 (E) 65

23. The shortest distance from the point $(0, 2)$ to a point on the parabola $y = x^2$ is closest to

- (A) 1.32 (B) 1.35 (C) 1.38 (D) 1.41 (E) 1.44

24. In the diagram, the circle has radius $\sqrt{50}$, $AB = 6$, $BC = 2$, and $\angle ABC = 90^\circ$. The distance from B to the centre of the circle is

- (A) $2\sqrt{7}$ (B) $2\sqrt{6}$ (C) $3\sqrt{3}$
(D) $\sqrt{26}$ (E) 5



25. A *Pythagorean triangle* is a right-angled triangle with integer side lengths a , b and c . The number of Pythagorean triangles with $a < b < c$ and $10 < a + b - c < 18$ is

- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13