The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents

The Ninth Annual
Small c Competition
for First and Second Year Students

Friday 02 October 2009

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open the Contest booklet until you are told to do so.

10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. This year, Tom-toms and Coleman stoves are again permitted, though perhaps for the last time. Protractors are also permitted, though contractors are not; if you can find a sub-contractor during this construction boom, by all means, use him/her.

11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.

100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.

101. On your response form, print your name, plan, and ID number.

110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.

111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.

1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.

1001. Diagrams are *not* drawn to scale. They are intended as aids only.

1010. Als u dit kunt lezen, spreekt u het Nederlands.

1011. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

1100. Please do not sing aloud while doing #19. Please note that motion sickness bags are not available.

Part A

1. The value of $2^3 + 1$ is

   (A) 3^2 **(B)** 2^4 **(C)** 3^3 **(D)** $2^4 - 1$ **(E)** 7

2. Which one of the following is equal to an even integer?

   (A) $2007^3 + 4$ **(B)** $2008^3 + 5$ **(C)** $2009^3 + 6$ **(D)** $2009^3 + 7$ **(E)** $2010^3 + 9$

3. The value of $\frac{2008}{2009} \cdot \frac{2009}{2010} \cdot \frac{2010}{2011} \cdot \frac{2011}{2012}$ is

   (A) $\frac{502}{503}$ **(B)** $\frac{1}{2012}$ **(C)** $\frac{1004}{1007}$ **(D)** $\frac{2008}{2011}$ **(E)** $\frac{2009}{2013}$

4. Amit starts with the number N, subtracts 15 and then divides by 17.
   Feridun starts with the number N, subtracts 17 and then divides by 15.
   The pair of Provosts profess to receiving the same answer.
   What is the value of N?

   (A) 288   **(B)** 15   **(C)** 17   **(D)** 2   **(E)** 32

5. Farmer Nicolai really likes Tetris! He has five fields, each formed from six squares of the same size. (Okay, so he has gone a bit beyond Tetris.) Which field has the smallest perimeter? (That is, which requires the least fencing?)

   (A)   **(B)**   **(C)**

   (D)   **(E)**
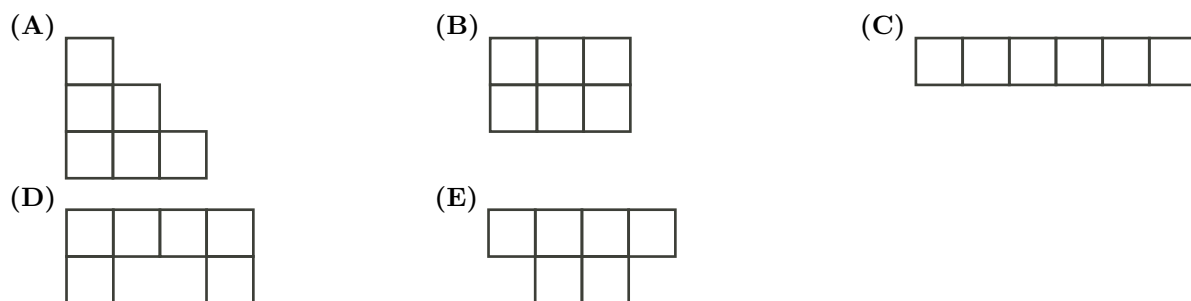
6. The expression $\sqrt{16^{16}}$ equals

   (A) 2^{32} **(B)** 8^8 **(C)** 4^8 **(D)** 2^{16} **(E)** 4^{12}

7. The numbers from 1 to 5 are to be written in a 5×5 grid so that each number appears exactly once in each row and in each column. Some of the numbers have already been entered. What number can go in the square marked x?

   (A) 5   **(B)** 4   **(C)** 3
   (D) 2   **(E)** 1

8. The number of different triangles in the figure, including all triangles of all sizes, is

   (A) 7   **(B)** 16   **(C)** 20
   (D) 22   **(E)** 24
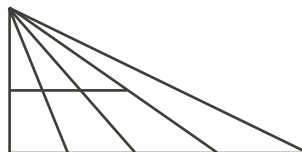
9. A sofa from the Comfy Lounge was left outside for two weeks this past July. Before being left outside, the mass of the sofa was M kg. As you probably remember, it rained constantly for those two weeks. At the end of its first week outside, its mass increased by 27 kg. Over the second week, its mass increased by 25% over the mass at the end of the first week. When MathSoc tried to move it after the two weeks, they found that its mass was $2M$ kg. What was the value of M?

(A) 81   (B) 48   (C) 45   (D) 36   (E) 50

10. The digit 5 is written between the digits of a two-digit number to form a three-digit number. This new number is 410 more than the original two-digit number. If the sum of the digits of this three-digit number is 12, what what is the difference between the digits of the two-digit number?

(A) 0   (B) 1   (C) 3   (D) 5   (E) 7

Part B

11. Riley is a poor starving student, but is mathematically astute. He notices that five suppers in residence cost the same as seven lunches. After one week of skipping supper most nights, he sees that five lunches and one supper cost \$62 in total. How much do 16 suppers cost?

(A) \$237   (B) \$197   (C) \$227   (D) \$217   (E) \$207

12. Two sides of a (non-degenerate) triangle have lengths 13 and 18. The third side also has an integer length, x. The number of possible values for x is

(A) 30   (B) 25   (C) 24   (D) 27   (E) 6

13. In the diagram, AB is parallel to CD, and $EG = FG$. Also, $\angle GFD = 10°$ and $\angle EGF = 80°$. Please, please, please, we implore you, what is the measure of angle BEG?

(A) 50°   (B) 60°   (C) 70°
(D) 80°   (E) 90°
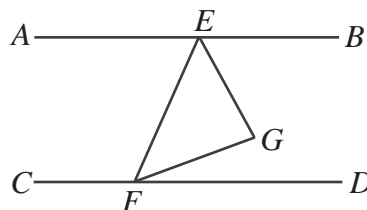
14. How many ordered pairs (a, b) of integers satisfy $a^2 + b^2 = 25$?

(A) 6   (B) 8   (C) 10   (D) 12   (E) 16

15. Three numbers x, y and z have a sum of 300 and have $y - x = z - y$. The product of the largest and smallest of the three numbers is 9711. The largest number is

(A) 113   (B) 117   (C) 123   (D) 119   (E) 127

16. How many real solutions does $x^2 + \sqrt{x^4 + 1} = 1$ have?

(A) 1   (B) 2   (C) 4   (D) 0   (E) 8

17. All together now! "The wheels on the bus go round and round, round and round, round and round..." Faculty of Math Dean Tom Coleman decided to ride the parking shuttle from UW's newest parking lot, located in Elmira. (It's the closest spot he could get!) He notices that, because of the exceptional skill of the driver, the bus travels at exactly 15π km/h, the wheels make exactly 5 revolutions per second, and the bus never slips or skids. What is the radius of one of the wheels, in metres?

(A) $\frac{5}{12}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{1}{6}\pi$ (E) $\frac{1}{9}\pi$

18. A regular n-gon is a polygon with n sides in which all of the sides are of equal length and all of the interior angles are of equal size. A regular 4-gon (better known as a square) has half as many diagonals as sides. What is the value of n if a regular n-gon has three times as many diagonals as edges?

(A) 6   (B) 8   (C) 9   (D) 10   (E) 12

19. Because of budget shortfalls in the new $250 million Quantum-Nano Centre, the benches are to be made out of leftover cabers from the Fergus Highland Games that are split in half and placed as in the diagram. If the radius of each of the lower logs is 20 cm and the radius of the upper log is 40 cm, what is the height of the bench, in centimetres?

(A) 50   (B) $40\sqrt{2}$ (C) 48
(D) $20\sqrt{10}$ (E) 60

20. The net shown in the diagram is to be folded into a regular icosahedron, which, being a regular icosahedron, has 20 faces. What is the product of the numbers on the triangular faces sharing an edge with the face labelled "1"?

(A) 1800   (B) 720   (C) 864
(D) 1536   (E) 768

Part C

21. Square $WXYZ$ has two of its vertices on the edges of equilateral $\triangle PQR$, as shown, so that $WP = 3$ and $XP = 2$. If the area of the square equals the area of $\triangle PQR$, the length of QR is closest to

(A) 3.94   (B) 3.98   (C) 3.90
(D) 4.06   (E) 4.02

22. The value of the infinite product $\dfrac{2^3 - 1}{2^3 + 1} \cdot \dfrac{3^3 - 1}{3^3 + 1} \cdot \dfrac{4^3 - 1}{4^3 + 1} \cdots = \lim\limits_{n \to \infty} \prod\limits_{k=2}^{n} \dfrac{k^3 - 1}{k^3 + 1}$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{2}{3}$ (D) $\frac{1}{\sqrt{3}}$ (E) $\frac{1}{2}$

23. If $3(2^a) + 5^b + 7^c + 11^d = 2008$, with $a, b, c,$ and d all non-negative integers, then $a + b + c + d$ equals

(A) 7   (B) 6   (C) 8   (D) 10   (E) 9

24. If x, y and z are real numbers with

$$\begin{aligned} x(x + y + z) &= 2 - yz \\ y(x + y + z) &= 4 - zx \\ z(x + y + z) &= 8 - xy \end{aligned}$$

what is the value of $|x + y + z|$?

(A) 2   (B) 2.5   (C) 3   (D) 3.5   (E) 4

25. The numbers 101, 104, 109, 116, 125, ... are of the form $a_n = 100 + n^2$, where $n = 1, 2, 3, 4, 5, \ldots$. For each n, let d_n be the greatest common divisor of a_n and a_{n+1}. What is the maximum value of d_n, as n ranges through the positive integers?

(A) 521   (B) 611   (C) 901   (D) 401   (E) 101