The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents

The Eleventh Annual
Small c Competition
for First and Second Year Students

Friday 23 September 2011

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open the Contest booklet until you are told to do so.

10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not; if you can find a sub-contractor during this construction boom, by all means, use him/her.

11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.

100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.

101. On your response form, print your name, plan, and ID number.

110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.

111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.

1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
    There is *no penalty* for an incorrect answer.
    Each unanswered question is worth 2, to a maximum of 20.

1001. Diagrams are *not* drawn to scale. They are intended as aids only.

1010. Als u dit kunt lezen, spreekt u het Nederlands.

1011. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

1100. Unfortunately, there is little room in this question booklet for you to sketch a new logo for the university.

1101. A 10 mark penalty will apply to any contestant overheard making disparaging remarks about the Toronto Maple Leafs.

1110. Bonus marks are available for the first student to successfully submit this cover page as the background image at http://www.uwaterloo.ca.

Part A

1. How many occurrences of the letter E are there in five copies of the logo to the right?

    (A) 5   **(B)** 10   **(C)** 15   **(D)** 20   **(E)** 25

2. The value of $\sqrt[\sqrt{9}]{27}$ is

    (A) $\dfrac{1}{3}$ **(B)** $\sqrt{3}$ **(C)** 3   **(D)** $3\sqrt{3}$ **(E)** 9

3. Four congruent squares are drawn as shown. A, B, C and D lie on a straight line and $AD = 9$. What is the shaded area?

    (A) 4   **(B)** 9   **(C)** 16   **(D)** 25   **(E)** 36

4. If $g(x) = h(x - 1)$ and $h(x) = x^2 - 1$, then the value of $g(4)$ is

    (A) 7   **(B)** 8   **(C)** 14   **(D)** 15   **(E)** 24

If you are a Montreal Canadiens fan and have made it this far, then CONGRATULATIONS!
You have exceeded expectations.

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5. How many non-zero digits are in the expansion of 2000^{11}?

    (A) 1   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** 5

6. During the royal visit, souvenir shops sold 65% more Kate bobblehead dolls than Will bobblehead dolls. What is the ratio of Will to Kate bobblehead dolls sold?

    (A) $7 : 13$ **(B)** $13 : 7$ **(C)** $13 : 20$ **(D)** $7 : 20$ **(E)** $20 : 33$

7. The average age of a group of teachers and students is 19. The average age of the teachers is 36. The average age of the students is 14. What is the ratio of teachers to students?

    (A) $\dfrac{7}{18}$ **(B)** $\dfrac{19}{50}$ **(C)** $\dfrac{5}{17}$ **(D)** $\dfrac{11}{18}$ **(E)** $\dfrac{7}{25}$

8. Kate has 500 different blouses, 300 different skirts and 400 different pairs of pants. Each day, she wears a blouse and either a skirt or pants. The only combination she will not wear is her purple blouse with her gold skirt. Beginning on the first day of the term, she decides to wear a different outfit every day as long as possible. How many different combinations does Kate wear before she has to repeat?

    (A) 320000   **(B)** 349999   **(C)** 350000   **(D)** 599999   **(E)** 600000

9. The curve shown models recent stock market activity. It consists of three semi-circles with diameters 6, 8 and 10 that meet at points on the line PQ. What is the length of the curve?

    (A) 6π **(B)** 12π **(C)** 20π **(D)** 24π **(E)** 50π

10. If $x \in \mathbb{R}$, then the maximum value of $\log_3 (80 - \sin x)$ is closest to

    (A) 3.98   **(B)** 3.99   **(C)** 4.00   **(D)** 4.01   **(E)** 4.02

Part B

11. What digit does ☐ represent if the sum of the three 3-digit numbers 31☐, 2☐5 and ☐84 is 12☐5?

    (A) 0   **(B)** 2   **(C)** 4   **(D)** 6   **(E)** 8

12. ABD is an equilateral triangle and BD is extended to C so that the area of triangle ABC is twice the area of ADC. The measure of angle BAC is

    (A) $90°$ **(B)** $120°$ **(C)** $100°$ **(D)** $105°$ **(E)** $72°$
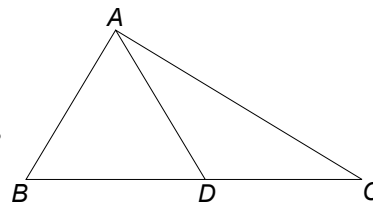
13. Twelve people live on an island. Each person is either a liar (never tells the truth) or a mathematician (always tells the truth). Luckily, at least one of them is a mathematician. At one island meeting, two people said, "There are exactly two liars among us". Four other people said, "There are exactly four liars among us". The remaining six people said, "There are exactly six liars among us." How many liars were there?

    (A) 2   **(B)** 4   **(C)** 6   **(D)** 8   **(E)** 10

14. If $\lim\limits_{x \to 1} \dfrac{x^2 + Ax + B}{x - 1} = 4$ then $A + B$ equals

    (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

15. Will has three red beads, three green beads and three white beads. In how may ways can these nine beads be placed in the configuration to the right so that no two similar-coloured beads are in the same row or column?
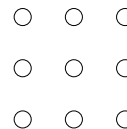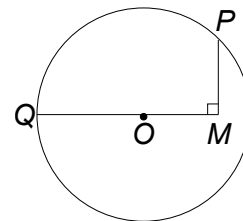
    (A) 3   **(B)** 6   **(C)** 9   **(D)** 12   **(E)** 24

16. What is the units digit of $\sum\limits_{k=1}^{2011} (k!)^3 = (1!)^3 + (2!)^3 + (3!)^3 + \ldots + (2011!)^3$?

    (Note: $m!$ is the product of the first m positive integers.)

    (A) 0   **(B)** 2   **(C)** 5   **(D)** 8   **(E)** 9

17. In the diagram, O is the centre of a circle with radius $a + 1$, $QM = 2a$ and MP is perpendicular to QM. What is the length of MP?

    (A) a **(B)** $2\sqrt{a}$ **(C)** $\sqrt{a}$ **(D)** $2\sqrt{a} + 2$ **(E)** $2\sqrt{a + 1}$

18. One Queen's commerce student randomly fills out a true/false test with 6 questions. Which of the following is most likely?

    (A) The student gets every question wrong.
    (B) The student gets exactly 1 or 2 questions right.
    (C) The student gets exactly 3 questions right.
    (D) The student gets exactly 4 questions right.
    (E) The student gets exactly 5 or 6 questions right.

19. The Winnipeg Jets will use 3-digit jersey numbers. The digit 0 will not be used. They will have the property that the middle digit is the mean (average) of the other two digits. How many jersey numbers are available?

    (A) 12   **(B)** 16   **(C)** 25   **(D)** 32   **(E)** 41

20. An equilateral triangle and a regular hexagon are inscribed in a circle which is itself inscribed in an equilateral triangle. L is the area of the large triangle, S is the area of the smaller triangle and H is the area of the hexagon. Which of these statements is true?

(A) $L = H + 3S$ (B) $H = LS$ (C) $H = \frac{1}{2}(L + S)$
(D) $H = L - S$ (E) $H = \sqrt{LS}$
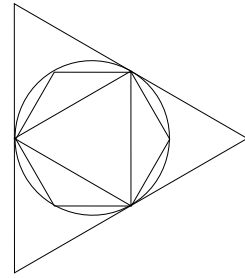
Part C

21. How many integers n are there such that $1 \leq n \leq 100$ and n^n is a perfect square?

(A) 99   (B) 55   (C) 50   (D) 10   (E) 5

22. Suppose that a, b, c and d are positive integers that satisfy the following equations.

$$
\begin{aligned}
abcd &= 5040 \\
ab + a + b + 1 &= 78 \\
bc + b + c + 1 &= 91
\end{aligned}
$$

What is the value of d?

(A) 35   (B) 28   (C) 21   (D) 14   (E) 7

23. Kate is given 99 distinct numbers. She has a machine that takes exactly two numbers a and b and outputs true if $a < b$ and false otherwise. She devises an algorithm that produces both the maximum and minimum of the 99 integers that uses the device at most n times. What is true about the smallest possible value of n?

(A) $n < 148$ (B) $148 \leq n < 164$ (C) $164 \leq n < 180$
(D) $180 \leq n < 196$ (E) $n \geq 196$

24. How many paths are there from A to D along segments of the diagram below that do not use any of the 7 segments more than once? For example, one path using 3 segments goes straight to D through B and C. A different path using 5 segments goes straight to D through B and C, then curves up back to A, and finally returns along the bottom curve back to D.

(A) 53   (B) 59   (C) 62   (D) 65   (E) 72
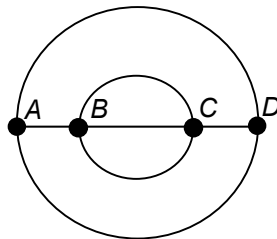
25. Will positions a cube so that one vertex is on a flat surface and the three vertices closest to but not on the surface, have distances of 2 cm, 3 cm and 4 cm to the surface. The length of the sides of the cube is closest to

(A) 5.4 cm   (B) 10.4 cm   (C) 5.2 cm   (D) 6.0 cm   (E) 9.0 cm