

*The Faculty of Mathematics at the University of Waterloo  
in association with  
The Centre for Education in Mathematics and Computing  
presents*

# The Twelfth Annual Small c Competition

for First and Second Year Students

Friday 21 September 2012

**Time:** 1 hour

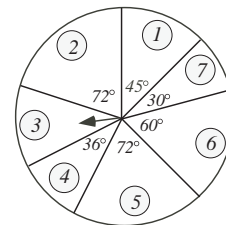
**Calculators are permitted.**

**Instructions:**

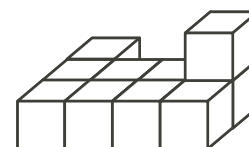
1. Do not open this booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not; if you can find a sub-contractor during this construction boom, by all means, use him/her.
11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
101. On your response form, print your name, plan, and ID number.
110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.
1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 20.
1001. Diagrams are *not* drawn to scale. They are intended as aids only.
1010. Als u dit kunt lezen, spreekt u het Nederlands.
1011. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.
1100. Unfortunately, there is little room in this question booklet for you to sketch a new logo for the university.
1101. A 10 mark penalty will apply to any contestant overheard making disparaging remarks about the Toronto Maple Leafs.
1110. Bonus marks are available for the first student to successfully submit this cover page as the background image at <http://www.uwaterloo.ca>.
1111. The only website you may use during the contest is [www.theonion.com](http://www.theonion.com).

## Part A

- The value of  $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$  is  
 (A)  $28^2$       (B)  $21^3$       (C)  $28^3$       (D)  $21^2$       (E)  $\pi^e$
- If  $f(x) = g(x - 2)$  and  $g(x) = -x^2$ , then the value of  $f(3)$  is  
 (A) 9      (B) -25      (C) -9      (D) 1      (E) -1
- Sue Silvester spins the circular spinner shown to decide how many times to pick on Will. What is the probability that the pointer will stop in the sector labelled ③?



- (A)  $\frac{1}{8}$       (B)  $\frac{1}{5}$       (C)  $\frac{1}{10}$       (D)  $\frac{1}{6}$       (E)  $\frac{1}{12}$
- Ten blocks have been glued together as shown. How many faces have been glued to another face?  
 (A) 12      (B) 24      (C) 10      (D) 22      (E) 20
- If  $\frac{1}{x} - \frac{x}{2} = 0$ , then the value of  $x^6$  is  
 (A) 8      (B) 6      (C)  $2\sqrt{2}$       (D)  $3\sqrt{2}$       (E) 64

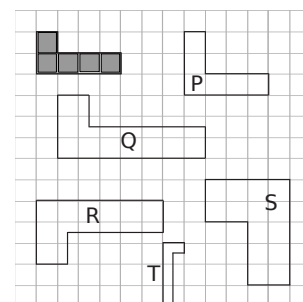


- If 10 kronkites make one spearo, and 35 spearos make one blard, how many blards do 25 kronkites make?  
 (A)  $\frac{5}{7}$       (B)  $\frac{1}{14}$       (C) 14      (D)  $\frac{2}{5}$       (E)  $\frac{5}{2}$
- In a set of ten numbers, the average of two of the numbers is 17 and the average of the other eight numbers is 12. The average of all ten numbers is  
 (A) 13      (B) 43.6      (C) 14.5      (D) 10      (E) 3.9

- How many odd positive integers are less than 47 and greater than or equal to 5?  
 (A) 20      (B) 21      (C) 22      (D) 23      (E) 24

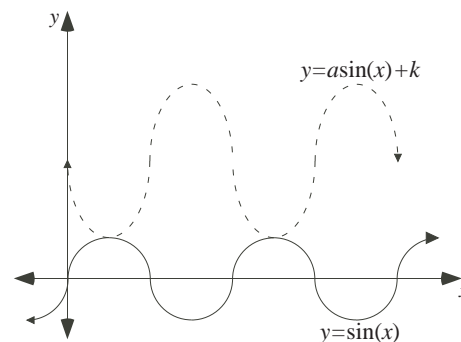
- The shaded shape represents Catniss' signature bookmark. Which other shape is a scaled and possibly reflected image of her signature bookmark?

- (A) P      (B) Q      (C) R      (D) S      (E) T



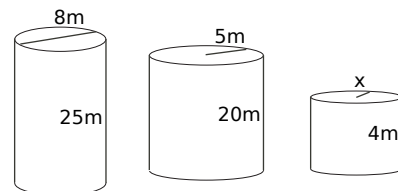
- The graph shown is a sketch of part of the functions  $y = \sin(x)$  and  $y = a \sin(x) + k$ . Which of the following must be true?

- (A)  $a = -1, k < 0$       (B)  $a > 0, k > 0$       (C)  $a < 0, k = 1$   
 (D)  $a < -1, k > 0$       (E)  $-1 < a < 0, k > 1$



## Part B

11. Lady Gogo's hair product is stored in three cylinders. One cylinder has a height of 25m and diameter of 8m. Another cylinder has a height of 20m and radius of 5m. A third cylinder with the smallest volume has a height of 4m and radius of  $x$ . The cylinder with the largest volume is the sum of the volumes of the other two cylinders. What is the value of  $x$ ?



- (A) 4m      (B) 4.7m      (C) 5m      (D) 5.3m      (E) 6m

12. On Monday, a group of students and teachers signed up for a field trip, with  $\frac{6}{7}$  of them being students. On Tuesday, another teacher signed up an additional 12 students. If  $\frac{7}{8}$  of the people signed up on Monday and Tuesday were students, how many people were there in total?

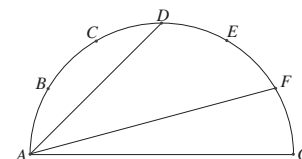
- (A) 48      (B) 90      (C) 42      (D) 45      (E) 27

13. The first  $k$  powers of two form the product  $2(2^2)(2^3) \cdots (2^k)$  which equals  $2^{120}$ . What is the value of the positive integer  $k$ ?

- (A) 15      (B) 5      (C) 120      (D) 6      (E) 114

14. Points  $A, B, C, D, E, F$ , and  $G$  are equally spaced around a semicircle with diameter  $AG$ . The measure of  $\angle DAF$  is

- (A)  $60^\circ$       (B)  $36^\circ$       (C)  $30^\circ$       (D)  $18^\circ$       (E)  $15^\circ$



15. Wolfgang Mario Christophoulus is unable to solve the Euro debt crisis so he starts playing with numbers instead. He writes down the list 0, 1, 1, 2, 4, 4, 5. Then he decides to change the numbers in the list in a way that will eventually make them all equal. He wants to do this in as few steps as possible, where a step consists of changing a single number in the list by increasing it or decreasing it by one. What is the minimum number of steps required?

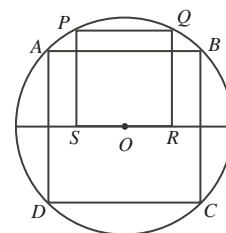
- (A) 10      (B) 11      (C) 12      (D) 13      (E) 17

16. Which of the following sums does not end in zero?

- (A)  $7^1 + 7^2 + 7^3 + 7^4 + 7^5 + 7^6 + 7^7 + 7^8$   
 (B)  $5^1 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7 + 5^8$   
 (C)  $9^1 + 9^2 + 9^3 + 9^4 + 9^5 + 9^6 + 9^7 + 9^8$   
 (D)  $2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8$   
 (E)  $6^1 + 6^2 + 6^3 + 6^4 + 6^5 + 6^6 + 6^7 + 6^8$

17. In the diagram shown,  $ABCD$  and  $PQRS$  are squares and  $O$  is the center of the circle. If the area of  $ABCD$  is  $a$  then the area of  $PQRS$  is

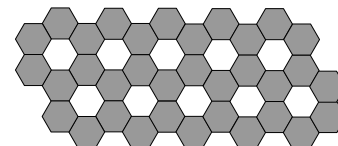
- (A)  $\frac{a}{2}$       (B)  $\frac{a}{3}$       (C)  $\frac{a}{4}$       (D)  $\frac{2a}{5}$       (E)  $\frac{\sqrt{2}a}{4}$



18. Given that  $f(x) = 3^{2x} - 12(3^x) + 27$ , the product of the  $x$ -intercepts of the graph of  $f(x)$  is

- (A) 2      (B) 3      (C) -12      (D) 27      (E) No  $x$ -intercepts

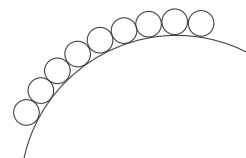
19. When Tom moved out, Katie retiled their (now her) *very, very, very large* rectangular floor using the pattern of *very, very, very small* congruent regular hexagons shown in the diagram. The portion of the floor that is white is closest to



- (A)  $\frac{1}{3}$       (B)  $\frac{1}{7}$       (C)  $\frac{1}{2}$       (D)  $\frac{1}{6}$       (E)  $\frac{1}{5}$

20. A large very round circle has radius 2012 units. Equally round circles of radius one unit are placed around the outside and tangent to the large circle with no overlapping. What is the maximum number of unit circles that can be used?

(A) 6320      (B) 6322      (C) 6324      (D) 12648      (E) 12641



## Part C

21. A sequence of six integers is an arrangement of the integers 1, 2, 3, 4, 5 and 6 in some order so that
- the sum of the integers 5 and 6 and all integers written down between them is 15,
  - the sum of the integers 1 and 2 and all integers written down between them is 12, and
  - the sum of the integers 4 and 5 and all integers written down between them is 21.

How many possible arrangements are there satisfying these conditions?

(A) 1      (B) 2      (C) 3      (D) 4      (E) 6

22. A circle is inscribed in triangle  $ABC$  tangent to the sides  $BC$ ,  $CA$ , and  $AB$  at points  $D$ ,  $E$ , and  $F$  respectively. If  $AB = 6$ ,  $AC = 14$  and  $BC = 16$ , then the length of  $CE$  is

(A) 4      (B) 6      (C) 8      (D) 10      (E) 12

23. Consider the sequence  $1^5, 2^5, 3^5, \dots$  where the  $k^{\text{th}}$  term equals  $k^5$  for all positive integers  $k$ . How many integers in the sequence are between the integers  $2^{36}$  and  $5^{17}$ ?

(A) 60      (B) 70      (C) 80      (D) 90      (E) 100

24. Suri is given nine objects with distinct weights. She is given a machine that takes two objects with weights  $a$  and  $b$  and produces true if  $a < b$  and false otherwise. She devises an algorithm that produces the second heaviest of the 9 objects using the machine at most  $n$  times. What is true about the smallest possible value of  $n$ ?

(A)  $n < 12$       (B)  $n = 12$       (C)  $n = 13$       (D)  $n = 14$       (E)  $n > 14$

25. What is the smallest positive integer  $d$  for which there exist six prime numbers  $n$ ,  $n + d$ ,  $n + 2d$ ,  $n + 3d$ ,  $n + 4d$ , and  $n + 5d$ ?

(A) 6      (B) 18      (C) 30      (D) 48      (E) 60