The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents

The Thirteenth Annual Small c Competition

for First and Second Year Students

Saturday 28 September 2013

Time: 1 hour
Calculators are permitted.
Instructions:

1. Do not open this booklet until you are told to do so.

10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not; if you can find a sub-contractor during this construction boom, by all means, use him/her.

11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.

100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.

101. On your response form, print your name, plan, and ID number.

110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.

111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.

1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.

1001. Diagrams are *not* drawn to scale. They are intended as aids only.

1010. Als u dit kunt lezen, spreekt u het Nederlands.

1011. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

1100. Unfortunately, there is little room in this question booklet for you to sketch a new logo for the university.

1001. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.

1110. Bonus: Successfully submit this cover page as the background image at `http://www.uwaterloo.ca`.

1111. The only website you may use during the contest is `www.theonion.com`.

10000. Data was scrambled in the CEMC's move to the sixth floor of the MC. Try and find the flipped bit above.

Part A

1. Which of the following is equal to half of 2^{2014} ?

   (A) 1^{1007} **(B)** 1^{2014} **(C)** 2014 **(D)** 2^{1007} **(E)** 2^{2013}

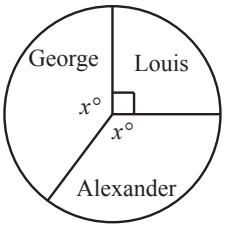
2. Four of the numbers in the following list share a common factor greater than one: 350, 363, 385, 420, 462. The number that does not share that factor is

   (A) 350 **(B)** 363 **(C)** 385 **(D)** 420 **(E)** 462

3. If $x + 2y = 13$ and $2x + y = 11$, what is the average of the numbers x and y ?

   (A) 4 **(B)** 8 **(C)** 12 **(D)** 3 **(E)** 5

4. The pie chart shows the breakdown of 2400 votes in an election. How many votes did Alexander receive ?



   (A) 800 **(B)** 900 **(C)** 1000 **(D)** 1200 **(E)** 1800

5. A flipped coin can land on heads or tails. Although unlikely, the only other possibility (on Earth) is that it lands standing on its edge. If the probability the coin lands on heads is $\frac{499}{1000}$ and the probability it lands on tails is also $\frac{499}{1000}$, what is the probability that it lands standing on its edge ?

   (A) $\frac{1}{250}$ **(B)** $\frac{1}{499}$ **(C)** $\frac{1}{500}$ **(D)** $\frac{1}{999}$ **(E)** $\frac{1}{1000}$

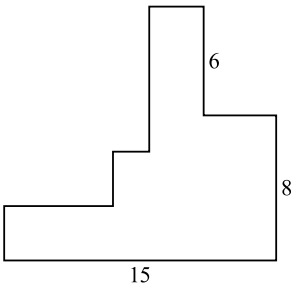
6. Two straight lines intersect at one point. All the points that are the same perpendicular distance from each of the two lines form

   (A) a parallelogram **(B)** a square **(C)** 1 straight line
   (D) a point **(E)** 2 perpendicular lines

7. If $\sqrt[4]{x} = 4$, then $\sqrt{x^4}$ equals

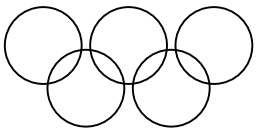
   (A) 1 **(B)** 4^4 **(C)** 4^6 **(D)** 4^8 **(E)** 4^{16}

8. In the figure shown, all adjacent sides are perpendicular. The perimeter of the figure is



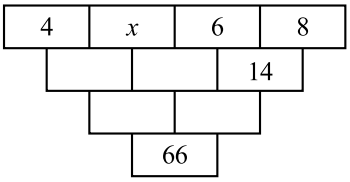
   (A) 39 **(B)** 58 **(C)** 116 **(D)** 201 **(E)** 290

9. Each circle has an area of 1 cm^2. The area of overlap between each pair of intersecting circles is $\frac{1}{8}$ cm^2. The total area of the region enclosed by the five circles is



   (A) $4\frac{1}{8}$ cm^2 **(B)** $4\frac{3}{8}$ cm^2 **(C)** $4\frac{7}{8}$ cm^2 **(D)** $4\frac{3}{4}$ cm^2 **(E)** $4\frac{1}{2}$ cm^2

10. Each number in this grid, after the first row, is obtained by adding two numbers directly above it. For example, $14 = 6 + 8$. The value of x is



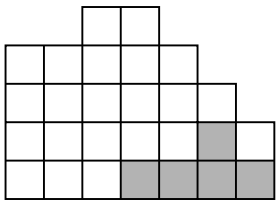
   (A) 10 **(B)** 12 **(C)** 14
   (D) 16 **(E)** 18

Part B

11. A farmer has both four-legged wombats and two-legged penguins. If the 100 animals have an average of $2\frac{1}{2}$ legs, then the number of penguins is

(A) 25 **(B)** 36 **(C)** 64 **(D)** 75 **(E)** 506

12. How many times can the five-square shape shown be uniquely found in the large shape ? The shape may be rotated and translated but not reflected.



(A) 22 **(B)** 26 **(C)** 28 **(D)** 29 **(E)** 35

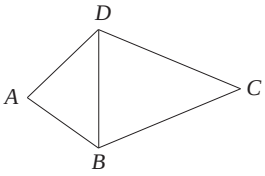
13. At Culombia Luke Vallige, the ratio of the number of red houses to the number of blue houses is $2 : 3$. Which of the following changes this ratio ?

(A) doubling the number of red houses and doubling the number of blue houses
(B) removing 2 red houses and removing 3 blue houses
(C) adding 4 more red houses and adding 6 more blue houses
(D) painting 5 red houses green and painting 5 blue houses green
(E) painting half the red houses yellow and painting half the blue houses yellow

14. If $x = 1$ is one solution of the equation $x^3 - 21x + 20 = 0$, then the sum of the other two solutions is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 20 **(E)** -21

15. In quadrilateral $ABCD$, the perimeter of $ABCD$ is 21, the perimeter of $\triangle DBC$ is 17, and the perimeter of $\triangle ABD$ is 12. Determine the length of DB.



(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

16. What is the minimum value of $4^{\log_2 x} - \log_x x^{4x} + 7$?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 7 **(E)** 8

17. The total number of distinct integral solutions to $(2x^2 - x - 6)(4x^2 + 17x - 15) = (4x^2 - 11x + 6)(x^2 + 6x + 5)$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** $\int$

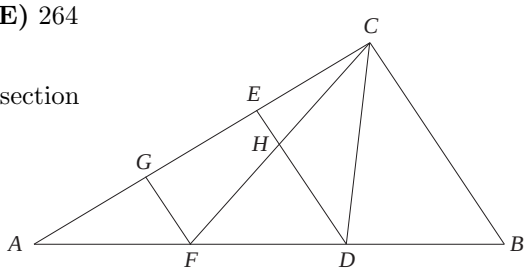
18. Suppose x, y and z, $x \neq 0$, are three consecutive terms in a geometric sequence that has a common ratio $r \neq 1$. If $12x$, $17y$ and $20z$ are three consecutive terms in an arithmetic sequence, then the sum of all possible values of r is

(A) $\frac{1}{2}$ **(B)** $-\frac{3}{5}$ **(C)** $\frac{6}{5}$ **(D)** $\frac{17}{10}$ **(E)** $-\frac{3}{10}$

19. How many integers n with $8 \leq n \leq 300$ cannot be written as the sum of two distinct elements of $\{3, 5, 8, 13, 21, 34, 55, 89, 144\}$?

(A) 257 **(B)** 258 **(C)** 261 **(D)** 263 **(E)** 264

20. In $\triangle ABC$, $AF = FD = DB$ and $BC \parallel DE \parallel FG$. H is the point of intersection of DE and CF. The ratio of the area of $\triangle CDH$ to the area of $\triangle CEH$ is



(A) 2 : 1 **(B)** 3 : 2 **(C)** 3 : 1
(D) 4 : 1 **(E)** 5 : 1

Part C

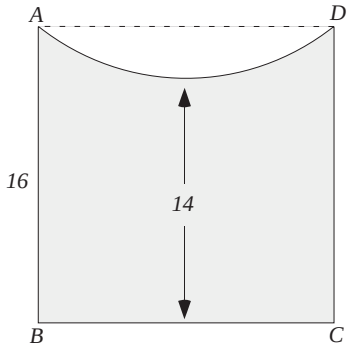
21. Determine the number of **ordered** real pairs (x, y) such that $xy = 28$ and $x^2 + y^2 = 65$.

    (A) 4 **(B)** 8 **(C)** 2 **(D)** 1 **(E)** 0

    Note: Proctors will not clarify whether or not $(1, 2)$ is the same as $(2, 1)$.

22. $ABCD$ is a square with side length 16. A segment of a circle has been cut out of one side, as shown, such that the distance from the lowest point of the arc to BC is 14. The area of the shaded region is closest to

    (A) 234.4 **(B)** 232.8 **(C)** 240.0 **(D)** 152.7 **(E)** 114.4



23. You are given the names of arguably the best five hockey players in the world in some order. You may repeatedly select and swap two names. Which of the following initial orders requires the most swaps to change the order to Bozak, Clarkson, Kessel, Lupul, Phaneuf ?

    (A) Phaneuf, Lupul, Kessel, Clarkson, Bozak
    (B) Lupul, Bozak, Phaneuf, Kessel, Clarkson
    (C) Clarkson, Lupul, Phaneuf, Bozak, Kessel
    (D) Lupul, Clarkson, Kessel, Bozak, Phaneuf
    (E) Kessel, Phaneuf, Lupul, Bozak, Clarkson

24. A five digit positive integer is called a *GeorgeAlexanderLouis number* if its digits are reversed when it is multiplied by four. How many GeorgeAlexanderLouis numbers are there ?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

25. Five distinct points are drawn on a circle. For each pair of points, Feridun flips a fair coin (that never lands on its side) and joins the two points with a straight line if and only if the coin comes up heads. The probability that Feridun does not draw a triangle with vertices among the five points is closest to

    (A) 0.3786 **(B)** 0.3787 **(C)** 0.3788 **(D)** 0.3789 **(E)** 0.3790