# The Faculty of Mathematics at the University of Waterloo in association with The Centre for Education in Mathematics and Computing presents

# The Fourteenth Annual Small c Competition

for First and Second Year Students

Friday 19 September 2014

Time: 1 hour

Calculators are permitted.

**Instructions:** 

- 1. Do not open this booklet until you are told to do so.
- 10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
- 11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
- 100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
- 101. On your response form, print your name, plan, and ID number.
- 110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
- 111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.
- 1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.

There is no penalty for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 20.

- 1001. Diagrams are *not* drawn to scale. They are intended as aids only.
- 1010. Als u dit kunt lezen, spreekt u het Nederlands.
- 1011. When your supervisor instructs you to begin, you will have sixty minutes of working time.
- 1100. Unfortunately, there is little room in this question booklet for you to sketch a new logo for the university.
- 1001. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
- 1110. The only website you may use during the contest is www.theonion.com.
- 1111. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
- 10000. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.

## Part A

1. What is the value of (9+8+7+6+5+4+3+2+1)+(1+2+3+4+5+6+7+8+9)? **(C)** 90 **(D)** 100 **(B)** 55 **(E)** 110 (A) 45

2. A tiny piece of a bacon-wrapped chocolate bar dipped in peanut butter contains 13 grams of fat which makes up 20% of your recommended daily intake. What is your total recommended daily intake of fat?

(C) 16.25 g (A) 2.6 g **(B)** 15.6 g **(D)** 26 g **(E)** 65 g

3. Answer ONE of the following two parts. They have the same numerical answer.

a) What is the maximum value of  $3\varepsilon + 2\nu$  subject to the following constraints?

$$\begin{array}{rcl} 2\varepsilon + \nu & \leq & 18 \\ 2\varepsilon + 3\nu & \leq & 42 \\ 3\varepsilon + \nu & \leq & 24 \\ \varepsilon & \geq & 0 \\ \nu & \geq & 0 \end{array}$$

b) What is the largest multiple of 11 less than 37?

**(C)** 33 (A) 11 **(B)** 22 (D) 44 **(E)** 55

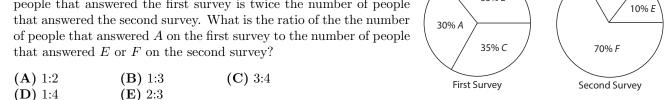
4. Rocket was very excited about the new coffee shops on campus. He had 3 cups of coffee on Monday and 4 cups of coffee on Tuesday. How many cups of coffee did he drink on Wednesday if the average over the three days is exactly 3 cups per day?

(A) 5 **(B)** 4 **(C)** 3 **(D)** 2 **(E)** 1

5. For which of the following values of a is  $a^2 - a^3$  the greatest? (Calculus is not recommended.)

(B)  $\frac{1}{4}$ (C)  $\frac{1}{2}$ (D)  $\frac{3}{4}$ (A)  $\frac{1}{8}$ **(E)** 1

6. The results of two surveys are shown on the right. The number of people that answered the first survey is twice the number of people that answered the second survey. What is the ratio of the the number of people that answered A on the first survey to the number of people that answered E or F on the second survey?



20% D

35% B

7. Gamora defines a sequence of numbers recursively by  $t_1 = 5$  and  $t_n = 3t_{n-1} - n$  for all natural numbers  $n \ge 2$ . The value of  $t_5$  is

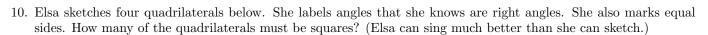
(A) 307 **(B)** 80 **(C)** 405 **(D)** 205 **(E)** 325

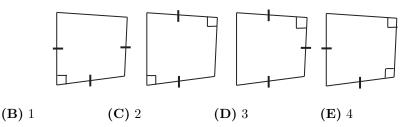
8. Nebula has 200 Canadian dollars (CAD). She converts some of that money to American dollars (USD) so that she has x CAD and x USD. If 1 USD = 1.05128 CAD, what is the value of x to the nearest cent?

(A) \$95.12 **(B)** \$97.50 (C) \$100.00 **(D)** \$102.50 **(E)** \$105.13

9. The number of real solutions to  $\ln(x^2) = \pi$  is

(A) -1**(B)** 0 **(C)** 2 **(D)** 4 (E)  $\infty$ 





#### Part B

**(A)** 0

11. What value is produced by the Scheme expression (max (- (+ 5 2) 1) (min (\* 3 0) 4)) when it is fully evaluated by DrRacket?

Notes for users of lame languages like Java, C, C++, C#, and especially lame languages like Visual Basic:

- i) Scheme uses prefix notation where  $(\odot \ a \ b)$  means  $a \odot b$  for some operation  $\odot$  and values a and b. For example, (max 3 4) evaluates to 4 and (\* 3 4) evaluates to 12.
- ii) Proctors will be available after the contest to talk about transferring Faculties.
- (A) 1
- **(B)** 3
- (C) 4
- (D) 6
- **(E)** 7
- 12. What is the sum of the first 41 digits of  $\frac{18}{11}$  expressed as a real number in decimal form? (The first 41 digits include those before the decimal point.)
  - $(\mathbf{A}) e$
- (B) 61
- **(C)** 181
- **(D)** 241
- **(E)** 361
- 13. In the diagram,  $\triangle ABC$  is a right isosceles triangle with AB = BC = 6. The perimeter of the rectangle DFEB is



- **(B)** 11
- **(C)** 11.5
- **(D)** 12
- **(E)** 12.5

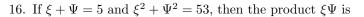
14. If  $\tan \theta = \frac{-3}{4}$ , then the value of  $(\sin \theta - \cos \theta)^2$  is

(A) 
$$\frac{49}{25}$$

- (B)  $\frac{-3}{5}$  (C)  $\frac{4}{5}$
- (D)  $\frac{25}{40}$
- (E)  $\frac{1}{25}$
- 15. A circle with centre Z has radius 5. Points X and Y lie on the circle so that XYZ is an equilateral triangle. How much longer is the arc on the circumference formed by the acute  $\angle XZY$  than the chord XY?



- (A)  $\frac{1}{6}(25\pi 30)$  (B)  $\frac{1}{3}(5\pi 15)$  (C)  $10\pi 5$
- **(D)**  $\frac{1}{3}(25\pi 15)$  **(E)**  $25\pi 5$



- (A) 6
- **(B)** -12
- (C) -14
- **(D)** 8
- **(E)** -24
- 17. In how many ways can a grouping of three  $1 \times 1$  tiles be placed in the grid shown where each  $1 \times 1$  tile is adjacent to at least one other tile in the group?

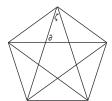


- **(B)** 14
- (C) 15
- **(D)** 16
- (E) 17



- 18. If  $2015\kappa\omega$  is a perfect square and  $\kappa$  and  $\omega$  are positive integers, what is the smallest possible value of  $\kappa + \omega$ ?
  - (A) 96
- **(B)** 49
- (C) 168
- **(D)** 408
- **(E)** 65





(A)  $120^{\circ}$ 

**(B)** 144°

**(C)** 116°

**(D)** 108°

**(E)**  $112^{\circ}$ 

20. Sven takes the ION rapid transit train at noon for a scheduled meeting with Olaf. If Sven travels at  $\rho$  km/h, he would arrive half an hour before their scheduled meeting time. If he travels at  $\psi$  km/h, he would arrive half an hour after their scheduled meeting time. In terms of  $\rho$  and  $\psi$ , at what speed, in km/h, should Sven travel to meet Olaf at the scheduled time?

(A) 
$$\frac{2\rho\psi}{\rho+\psi}$$

(B) 
$$\frac{\rho + \psi}{2}$$

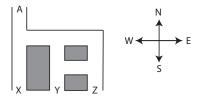
(C) 
$$\frac{\rho\psi}{2(\rho+\psi)}$$

(D) 
$$\frac{\rho\psi}{\rho-\nu}$$

(A) 
$$\frac{2\rho\psi}{\rho+\psi}$$
 (B)  $\frac{\rho+\psi}{2}$  (C)  $\frac{\rho\psi}{2(\rho+\psi)}$  (D)  $\frac{\rho\psi}{\rho-\psi}$  (E)  $\frac{3(\rho+\psi)}{2(\rho-\psi)}$ 

### Part C

21. Fob Rord begins at A in the map below. He always moves south or east. At each intersection where he has a choice of continuing south or east, he flips a fair coin to make this choice. He is done his journey when he arrives at X, Y or Z. What is the probability that he ends at Z?



(A)  $\frac{1}{8}$ 

(B)  $\frac{2}{8}$ 

(C)  $\frac{3}{8}$ 

(D)  $\frac{4}{8}$ 

(E)  $\frac{5}{8}$ 

22. PQRS is a square. T is the midpoint of PQ and U is the midpoint of QR. What fraction of PQRS has been shaded?

(A)  $\frac{3}{5}$ 

(B)  $\frac{5}{12}$  (C)  $\frac{7}{10}$  (D)  $\frac{7}{12}$ 

(E)  $\frac{5}{0}$ 

23. What is the value of the following product?  $\prod_{m=1}^{49} \left( \frac{\frac{1}{2m} - \frac{1}{2m+1}}{\frac{1}{2m+1} - \frac{1}{2m+2}} \right)$ 

Note:  $\Pi$ -notation is the same as  $\Sigma$ -notation except it represents a product instead of a sum. For example:

$$\prod_{m=1}^{3} (m^2) = (1^2)(2^2)(3^2) = 36.$$

(A) 0.01

**(B)** 0.02

(C)  $\frac{1}{2}$ 

**(D)** 2

**(E)** 50

24. Let S be the sum of all 2-digit integers having exactly 12 positive divisors. What is the sum of the squares of the digits in S?

(A) 101

**(B)** 14

(C) 20

**(D)** 41

**(E)** 34

25. Star-Lord declares the positive integers from 1 to 279 nice. He declares the 1735 positive integers from 280 to 2014 mean. How many ordered triples (a, b, c) are there such that

•  $a, b, c \in \mathbb{Z}$ , and

•  $1 \le a, b, c \le 2014$ , and

• 2014 divides a + b + c, and

• a, b and c are either all nice or all mean?

(A) 2603995

**(B)** 2604001

(C) 2604007

**(D)** 2604013

**(E)** 2604019