

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents*

The Sixteenth Annual Small c Competition

for First and Second Year Students

Friday 23 September 2016

Time: 1 hour

Calculators are permitted.

Instructions:

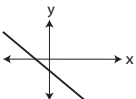
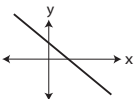
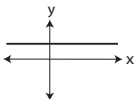
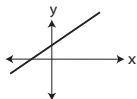
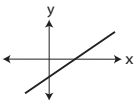
1. Do not open this booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.
110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.
1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
1001. Diagrams are *not* drawn to scale. They are intended as aids only.
1010. Als u dit kunt lezen, spreekt u het Nederlands.
1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.
1100. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
1001. The only website you may use during the contest is www.theonion.com.
1110. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
1111. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.
10000. Hint: The answer to at least one question is **B**.
10001. If you finish early, please Brexit the room quietly.

Part A

1. If $u = 18$ and $v = 75$, then $\sqrt{\frac{2u}{3v}}$ equals

- (A) $\frac{2}{15}$ (B) $\frac{2}{25}$ (C) $\frac{2}{5}$ (D) $\frac{4}{9}$ (E) $\frac{4}{3}$

2. A reasonable sketch of the line $y = -\frac{2}{3}x + 5$ is

- (A)  (B)  (C)  (D)  (E) 

3. $2^2 4^4 8^8$ equals

- (A) 64^{14} (B) 64^{64} (C) 2^{34} (D) 2^{19} (E) 2^{384}

4. How many positive integers are less than 20.17×5 ?

- (A) 99 (B) 100 (C) 101 (D) 102 (E) 103

5. The top four scorers on the Small c contest for the past 15 years were given a cash prize. The chart below shows how each of these students chose to spend their money. What is the difference between the percentage of students who chose to spend their prize on video games and the percentage of those who chose to spend their prize money on textbooks? (Note that |||| means five students so, for example, ||||| is a tally of seven students.)

Expense	Number of Students
Video Games	 
Fast Food	
Textbooks	
Music	

- (A) 10 (B) 12 (C) 13 (D) 15 (E) 25

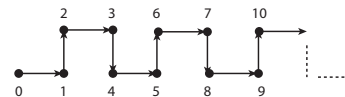
6. A tire has a radius of 23 cm. The distance between Kitchener and Toronto is about 110 km. Approximately how many complete rotations will the tire make in a round trip?






- (A) 478 261 (B) 304 470 (C) 152 235 (D) 74 118 (E) 4873

7. Which of the following triples cannot be the lengths of the sides of a triangle?

- (A) (2, 5, 6) (B) (3, 4, 5) (C) (5, 12, 13) (D) (6, 8, 8) (E) (2, 3, 6)

8. The “right,up,right,down” pattern shown to the right continues indefinitely. Which arrow configuration goes from point 2014 to point 2016? (*This is not a rigorous definition of a pattern but after hours of deliberation, the Small c committee concluded that all reasonable people will see the same pattern.*)



- (A)  (B)  (C)  (D)  (E) 

9. When Arcanine put 12 marbles in her bag, it was $\frac{1}{2}$ full. When Blastoise put 9 marbles in his bag, it was $\frac{1}{3}$ full. When Charizard put 7 marbles in her bag, it was $\frac{1}{4}$ full. When Dragonite put 16 marbles in his bag, it was $\frac{2}{3}$ full. When Electabuzz put 18 marbles in her bag, it was $\frac{3}{4}$ full. Whose bag contains the most marbles when full?

- (A) Arcanine (B) Blastoise (C) Charizard (D) Dragonite (E) Electabuzz

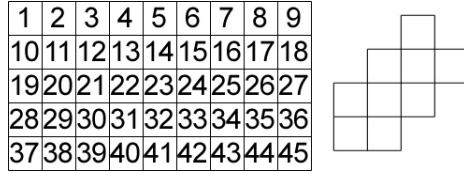
10. If $f(x) = x^2 - 9x + q$, $f(w) = -16$, $w = v$, $v = u$, $u = t$, $t = s$, $s = r$, and $r = q$, then the value of $f(-2)$ is

- (A) 26 (B) 4 (C) -4 (D) 6 (E) -10

Part C

21. Following his break-up with Taylor, Tom decided to remodel his kitchen. He numbered the tiles on the floor as shown below and bought an irregular island in the shape given. He placed the island (without rotating it) so that the sum of the covered prime numbers was as small as possible. What is this sum?

- (A) 17 (B) 20 (C) 41 (D) 14 (E) 47



22. Let x and y be positive integers where $x > 1$. If both $44y + 865$ and $53y + 1038$ are multiples of x , what is x ?

- (A) 5 (B) 17 (C) 53 (D) 143 (E) 173

23. A square-based pyramid of height 3 cm and base length 2 cm is set on its square base. Water is poured through an infinitesimally small hole in the top of the pyramid to a height of 1 cm measured from the base, and then the hole is sealed. If the pyramid is turned upside-down, what will be the new height of the water?

Reminder: The volume of a square-based pyramid of height $\Xi > 1$ and base length $\Psi > 1$ is approximately

$$\left| \sqrt{(9 \cdot \Psi \cdot \left(\frac{1}{3^3}\right) \cdot \Xi \cdot \sin\left(\frac{\pi}{2}\right) \cdot \Psi)^2 + 0.00000000000000000000000000000001} \right|.$$

- (A) $\sqrt[3]{19}$ (B) 2 (C) 1 (D) $\sqrt[3]{25}$ (E) $\frac{76}{27}$

24. How many different 8-digit numbers can be formed using two 1s, two 2s, two 3s, and two 4s such that no two adjacent digits are the same?

- (A) 720 (B) 792 (C) 840 (D) 864 (E) 1008

25. A box contains three white balls and two blue balls. Hilllllly draws a ball from the box with replacement. If she draws a white ball, she wins a dollar and if she draws a blue ball, she wins two dollars. She can keep the money if and only if the total amount she has won is ever exactly \$100. If p is the approximate probability that Hilllllly keeps the money, what is the 32^{nd} digit after the decimal point in p ?

Hint: This probability is less than the probability that there will be an NHL hockey team in Las Vegas in 2017.

- (A) 4 (B) 1 (C) 6 (D) 7 (E) 2