

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents*

The Seventeenth Annual Small c Competition

for First and Second Year Students

Friday 22 September 2017

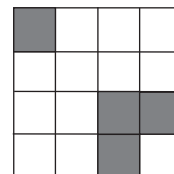
Time: 1 hour

Calculators are permitted.

Instructions:

1. Do not open this booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.
110. This is a multiple choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.
1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
1001. Diagrams are *not* drawn to scale. They are intended as aids only.
1010. Als u dit kunt lezen, spreekt u het Nederlands.
1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.
1100. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
1001. The only website you may use during the contest is www.theonion.com.
1110. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
1111. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.
10000. Hint: The answer to at least one question is **B**.
10001. If you finish early, please Brexit the room quietly.
10010. Don't give up. As shown by the U.S. election, anyone can win this thing.

Part A



1. A square is divided into small identical squares as shown. What percentage of the larger square is shaded?

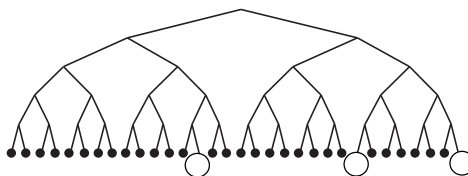
(A) 12.5% (B) 25% (C) 50% (D) 75% (E) 87.5%

2. If July 3, 2091 is a Tuesday, what day of the week is July 28, 2091?

(A) Tuesday (B) Friday (C) Saturday (D) Sunday (E) Monday

3. How many dots are there in the diagram below? Every dot is a circle but not every circle is a dot.

(A) 5 (B) 13 (C) 29 (D) 61 (E) 125

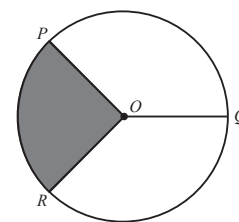


4. The value of $\sqrt{3\sqrt{3\sqrt{9}}}$ is

(A) $\sqrt{3}$ (B) 3 (C) $3\sqrt{3}$ (D) 9 (E) $9\sqrt{3}$

5. A malfunctioning homemade solar eclipse viewer produces the diagram shown in which $\angle POQ = \angle ROQ = 135^\circ$. What fraction of the circle with centre O is shaded?

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$



6. For all real numbers x except $x = -3$ and $x = 0$, if $\frac{1}{x+3} + \xi = \frac{1}{x}$, then ξ equals

(A) $\frac{3}{x(x+3)}$ (B) -3 (C) $-\frac{1}{3}$ (D) $\frac{2}{x+3}$ (E) $\frac{0}{-3}$

7. What is the greatest number of dice with dimensions $2 \text{ nm} \times 2 \text{ nm} \times 2 \text{ nm}$ that can fit in a rectangular box with dimensions $10 \text{ nm} \times 10 \text{ nm} \times 10 \text{ nm}$? (These are not fluffy stuffed dice. They are rock solid and do not squish.)

(A) 25 (B) 12 (C) 125 (D) 250 (E) 100

8. Four distinct lines are drawn in pencil crayon. What is the maximum possible number of intersection points? (TIL *pencil crayon* is “Canadian” for coloUred pencil.)

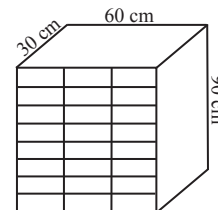
(A) 2 (B) 3 (C) 4 (D) 6 (E) 8

9. The product of two consecutive positive integers is 12 less than the square of the greater integer. What is the sum of these two integers?

(A) 13 (B) 23 (C) 35 (D) 57 (E) 99

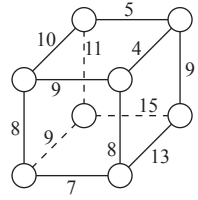
10. A storage unit consisting of smaller sections meant to (awkwardly) hold pink ties is shown. It is made of infinitesimally thin but surprisingly strong material. Each section is a rectangular prism open only at the front. How many square METRES of the material is needed to make the entire storage unit? Don't forget the **B**ack of the storage unit.

(A) 2.790 (B) 2.700 (C) 0.162 (D) 1.980 (E) 3.240



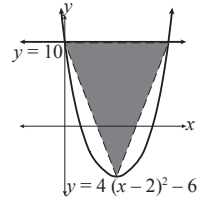
Part B

11. The numbers from 1 to 8 are each written in invisible ink at a vertex of the cube shown. For each edge, the sum of the numbers at its end vertices is shown in black ink. What is the sum of the numbers at the four vertices of the back of the cube?



- (A) 16 (B) 17 (C) 19 (D) 20 (E) 22

12. What is the area of the shaded triangle in the diagram? Its base lies along the line with equation $y = 10$ and it shares a vertex with the parabola with equation $y = 4(x - 2)^2 - 6$.



- (A) 32 (B) 16 (C) 96 (D) 64 (E) 20

13. Let $a_{n+2} = a_{n+1} + a_n$ for all positive integers n . If $a_6 = 2$ and $a_8 = 7$, then $a_1 + a_{10}$ equals

- (A) 28 (B) 60 (C) 26 (D) 10 (E) 13

14. If $n = p \cdot q \cdot r$ where p , q and r are distinct prime numbers and $n < 1000$, then the largest possible value of n is

- (A) 986 (B) 987 (C) 994 (D) 996 (E) 999

15. Feridun rides an ION train which is moving along a straight track at a constant velocity. He observes posts which are evenly spaced at the side of the track. When seated, he sees 10 posts per minute and when walking towards the front of the train he sees 11 posts per minute. What is the ratio of the train's velocity to Feridun's walking velocity?

- (A) 9 : 1 (B) 11 : 1 (C) 10 : 1 (D) 1 : 10 (E) 1 : 11

16. If $x + y = 3$ and $3x^2 + 4xy + y^2 = 12$. Then $6x + 2y$ is

- (A) 6 (B) 4 (C) 3 (D) 8 (E) 7

17. How many four-digit numbers, $ABCD$, are there such that AB and CD are two-digit prime numbers and $AB < CD$?

- (A) 210 (B) 441 (C) 525 (D) 300 (E) 253

18. Integers a and b are both multiples of integer c . How many of the following four statements must be true?

- $5ab$ is a multiple of c
- cba is a multiple of ac^2
- $a - c$ is a multiple of b
- $2a - 3b$ is a multiple of c

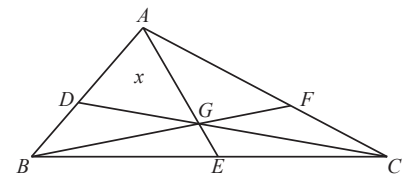
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

19. The University of Kitcherloo is going to build k Engineering buildings. In each building, the number of offices is to be between 1 and 2017, inclusive. What is the minimum possible value of k that guarantees that four of these buildings will have the same number of offices?

- (A) 2018 (B) 4035 (C) 6052 (D) 8069 (E) 10086

20. In $\triangle ABC$, $AD : DB = 5 : 3$, $BE : EC = 6 : 5$, and $AF : FC = 2 : 1$. If the area of $\triangle GFC$ is 220, then what is the area of $\triangle BGE$?

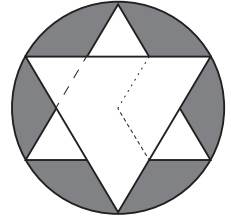
(Yes, we know there is a "random" variable x in the diagram. We also know that it does not lie inside $\triangle BGE$.)



- (A) 210 (B) 216 (C) 219 (D) 220 (E) 215

Part C

21. Two superimposed equilateral triangles with side length side 15 and parallel bases are inscribed in a circle as shown. Mysterious dashed lines also appear in the diagram. What is the shaded area?



(The units don't matter but if this bothers you, assume the area is measured in pari. Recall: 1 pari equals 2 lourak, 4 sangam, 8 loukhai, 16 loushal, or 32 tong.)

(A) $60(\pi - \sqrt{3})$ (B) $65(\pi - \sqrt{3})$ (C) $75(\pi - \sqrt{3})$ (D) $80(\pi - \sqrt{3})$ (E) $85(\pi - \sqrt{3})$

22. What is the remainder when $7^{700}2^{200} + 18400070000560004200028000140002018 - 14^{2017}$ is divided by 14?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

23. Consider the sum $360000 + 180002 + 90003 + 45003.5 + \dots$ where the sum begins with the number 360000, and then each subsequent term in the sum is determined by dividing the preceding term by 2 and then adding 2. If the sum contains 100 terms, then the value of the sum is closest to

(A) 720388 (B) 720392 (C) 720396 (D) 720400 (E) 720404

24. The size of the set $\left\{ (x, y) \in \mathbb{Z} \times \mathbb{Z} : \frac{1}{x} + \frac{x}{y} + \frac{253}{xy} = 1 \right\}$ is

(A) 0 (B) 3 (C) 4 (D) 7 (E) 8

25. A pyramid has base $WXYZ$ which is a unit square and apex V such that the slant heights are $VX = VY = VZ = VW = 2$. The midpoint of VX is T and the midpoint of VZ is U . One plane passes through the points T , U and W intersecting VY at R . A second plane passes through the points T , U and Y intersecting VW at S . The volume of the region bounded by the eight triangles RST , RSU , RTY , RUY , STW , SUW , TWY , and UWY is closest to

(A) 0.275 (B) 0.276 (C) 0.277 (D) 0.278 (E) 0.279

