

*The Faculty of Mathematics at the University of Waterloo
in association with
The Centre for Education in Mathematics and Computing
presents*

The Eighteenth Annual Small c Competition

for First and Second Year Students

Friday 28 September 2018

Time: 1 hour

Calculators are permitted.

Instructions:

1. Do not open this booklet until you are told to do so.
10. You may use slide rules, abaci, rulers, compasses and paper for rough work. You may also use log tables; log cabins are not permitted. Protractors are also permitted, though contractors are not.
11. By Faculty policy, only fourth-year students are allowed to use scissors. (Of course, they can't run with them.) Thus, there are no scissors allowed on the Small c.
100. Any contestant carrying an Elongated Pentagonal Orthocupolarotunda must register it with a proctor.
101. You must **print your name and ID number on the response form**. No other information is needed but answers to the contest questions are highly recommended.
110. This is a multiple choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate bubble on the response form.
111. In the past, your response form was read only by a *dumb human*, who had undergone rigorous training in order to be able to recognize the letters **A** through **E**. Due to labour unrest, this year, the dumb humans have been replaced by even dumber machines.
1000. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
1001. Diagrams are *not* drawn to scale. They are intended as aids only.
1010. Als u dit kunt lezen, spreekt u het Nederlands.
1011. When a proctor instructs you to begin, you will have 111100 minutes of working time.
1100. Anyone overheard making a joke about the Toronto Maple Leafs will be immediately removed from the premises.
1001. The only website you may use during the contest is www.theonion.com.
1110. Data was scrambled during construction in the MC building. Try and find the flipped bit above.
1111. Turn off and put away your cell phones, tablets, laptops, desktops, satellites and quantum computers.
10000. Hint: The answer to at least one question is **B**.
10001. If you finish early, please Brexit the room quietly.
10010. Don't give up. As shown by the U.S. election, anyone can win this thing.
10011. One bonus mark will be awarded to any contestant that gets Question 1 wrong and Question 25 right.

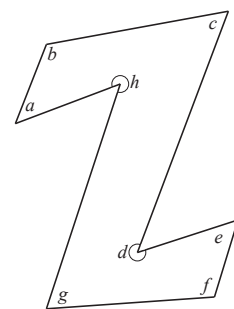
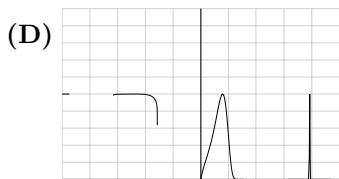
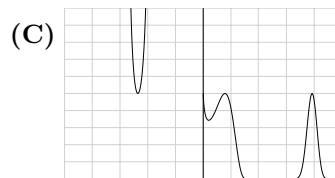
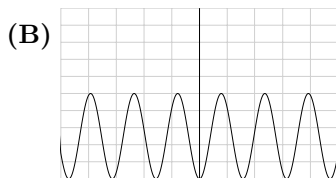
Part B

11. The third prime number larger than 49 is
 (A) 52 (B) 53 (C) 57 (D) 59 (E) 61
12. When $\frac{111457}{999000}$ is expressed as a decimal, what is the 2018th digit after the decimal point?
 (A) 1 (B) 5 (C) 6 (D) 8 (E) 9
13. Suppose K , M , and N are different digits and the following sum holds (i.e. is not fake news).

$$\begin{array}{r} K K \\ M M \\ + N N \\ \hline N M K \end{array}$$

The digit M is

- (A) 8 (B) 9 (C) 7 (D) 1 (E) 6
14. What is the area bounded by the lines $y = 20x$, $y = 19x$ and $x = 2018$?
 (A) $\frac{2018^2}{2}$ (B) 2018 (C) 2018^2 (D) $\frac{2018 \times 2020}{2}$ (E) $\frac{2018}{2}$
15. Sue Perlazy used FooPlot[©] to graph $y = (\sin x)^2$, $y = (\sin x)^{(e^x)}$, $y = (\sin x)^{(\log x)}$, $y = (\sin x)^{(\cos x)}$ and $y = (\sin x)^x$. The five graphs displayed by FooPlot[©] are shown below. Which graph was displayed for $y = (\sin x)^{(\cos x)}$?



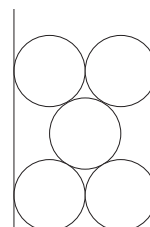
16. What is the sum of the measures of the eight interior angles a, b, c, d, e, f, g, h of the figure shown?
 (A) 1080° (B) 900° (C) 720° (D) 840° (E) 960°

17. When $ax^2 + \text{duck}$, where a and duck are real numbers, is substituted for x in the expression $2x^2 - 3x + 4$, the result is $8x^4 + 34x^2 + 39$. A possible value for $a + \text{duck}$ is

- (A) $\frac{-11}{2}$ (B) -7 (C) $\frac{-3}{2}$ (D) 3 (E) $\frac{3}{2}$

18. Circular discs of radius R are placed in a rectangular column of width $4R$ in an alternating pattern of two then one in the middle as in the diagram. What is the height of the column of discs when there are 20 discs in the column?

- (A) $(2 + 11\sqrt{3})R$ (B) $(2 + 12\sqrt{3})R$ (C) $(2 + 13\sqrt{3})R$
 (D) $(1 + 12\sqrt{3})R$ (E) $(3 + 12\sqrt{3})R$

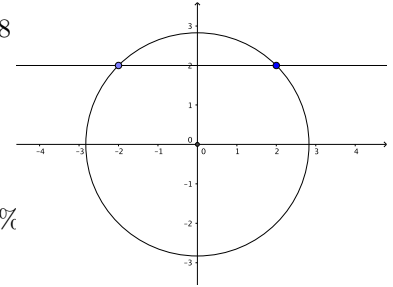


19. One clear afternoon, geese on campus could see a distance of 10 km. One goose was perched on top of the Dana Porter Library. A second goose was perched at the same height but 500 m away on the tall mysterious smoke stack. Suddenly, both geese started flying in opposite directions at different but constant speeds. While flying, they could see each other (using eyes on the backs of their heads) for 20 minutes. If one goose was flying at 12 km/h, how fast was the other goose flying, in km/h?

(A) 16.5 (B) 17 (C) 40.5 (D) 42 (E) 18

20. A circle has its centre at the origin. A horizontal line is drawn through the circle and intersects the circle at two points, $(-2, 2)$ and $(2, 2)$. To the nearest integer, what percentage of the circle's area is above the line?

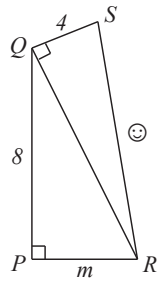
(A) 12% (B) 11% (C) 8% (D) 10% (E) 9%



Part C

21. Triangles PQR and QRS are right-angled, as shown. For how many different positive integer values of m is \ominus also a positive integer?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4



22. Justin, Sophie, Xavier, Ella-Grace, and Hadrien are splitting a collection of 18 identical \ominus emojis. If each person must have at least 2 \ominus emojis and no person can have more than 6 \ominus emojis how many ways can they be shared?

(A) 320 (B) 260 (C) 315 (D) 290 (E) 350

23. The *digital root* of a non-negative integer is the (single digit) value obtained by an iterative process of summing digits, on each iteration using the result from the previous iteration to compute a digit sum. The process continues until a single-digit number is reached. (**Acknowledgement: stolen from Wikipedia so it must be right.**)

A \ominus emoji number is an integer $n > 1$ such that the digital root of n^m is equal for all integers $m > 1$.

For example, 18 is a \ominus emoji number because:

$$18^2 = 324 \text{ and } 3 + 2 + 4 = 9$$

$$18^3 = 5832 \text{ and } 5 + 8 + 3 + 2 = 18 \text{ and } 1 + 8 = 9$$

$$18^4 = 104976 \text{ and } 1 + 0 + 4 + 9 + 7 + 6 = 27 \text{ and } 2 + 7 = 9$$

$$18^5 = 1889568 \text{ and } 1 + 8 + 8 + 9 + 5 + 6 + 8 = 45$$

$$\text{and } 4 + 5 = 9$$

⋮

How many \ominus emoji numbers are there less than or equal to 2018?

(A) 892 (B) 893 (C) 894 (D) 895 (E) 896

24. Two cuts are made at randomly chosen points on a line segment to form three line segments. What is the probability that these three line segments can be used to form a triangle? (Hint: It has nothing to do with \ominus emojis.)

(A) $\frac{1}{3}$ (B) $\frac{1}{8}$ (C) $\frac{2}{3}$ (D) $\frac{1}{4}$ (E) $\frac{3}{4}$

25. How many ways are there to place 10 \ominus emojis in 10 different squares of a 5×5 grid so that each column contains two \ominus emojis and each row contains two \ominus emojis?

(A) 1860 (B) 1920 (C) 1980 (D) 2040 (E) 2100