

1: Express each of the following complex numbers in cartesian form.

(a) $(2 + i)(3 + 2i) - (5 + 3i)$

(b) $\frac{1}{(\sqrt{2} + i)(1 + i\sqrt{2})}$

(c) $\frac{(1 + 2i)^3}{(3 + i)^2}$

2: Solve each of the following equations for $z \in \mathbf{C}$. Express your answers in cartesian form.

(a) $\frac{z + 1}{z + i} = 3 + i$

(b) $z^2 = \bar{z}$

(c) $z^2 = 4 + 3i$

3: Solve the following pairs of equations for $z, w \in \mathbf{C}$.

(a) $z + iw = 2$
 $iz + 2w = 3$

(b) $iz + (1 + i)w = -3 + i$
 $(2 + i)z + (3 - 2i)w = 4i$

4: Draw a picture of each of the following subsets of the plane.

(a) $\{z \in \mathbf{C} \mid 1 < |z - 1| \leq \sqrt{5}\}$

(b) $\{z \in \mathbf{C} \mid |z - 2i| = |z - 4|\}$

(c) $\{z \in \mathbf{C} \mid |z| + |z - 4| = 8\}$

5: For real numbers x and y , we define

$$e^{x+iy} = e^x \cos y + i e^x \sin y.$$

For a complex number z , we define

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \quad \text{and} \quad \sin z = \frac{e^{iz} - e^{-iz}}{2i}.$$

(a) Show that for all $z, w \in \mathbf{C}$ we have $e^{z+w} = e^z e^w$.

(b) Show that for all $z, w \in \mathbf{C}$ we have $\sin(z + w) = \sin z \cos w + \cos z \sin w$.

(c) Solve $e^z = 1 + i\sqrt{3}$ for $z \in \mathbf{C}$.

(d) Solve $\sin z = i$ for $z \in \mathbf{C}$.