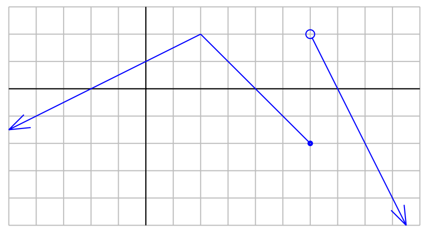
MATH 138 Calculus 2, Assignment 1 Due Mon Sept 23, 11:00 pm

1: Let $g(x)=\int_0^x f(t)\,dt$ where $f(t)$ is the function whose graph is shown below. Sketch the graph of $y=g(x)$ showing all intercepts, all local maxima and minima, and all points of inflection.



2: (a) Let $f(x)=\dfrac{8x}{2^{3x}}$. Approximate the integral $\int_0^2 f(x)\,dx$ using the Riemann sum for $f(x)$ which uses the right endpoints of 6 equal-sized subintervals.

(b) Let $f(x)=\dfrac{1}{x}$. Approximate the integral $\int_{1/5}^{13/5} f(x)\,dx$ using the Riemann sum for $f(x)$ which uses the midpoints of 6 equal-sized subintervals.

3: (a) Evaluate $\int_1^3 x^3-3x\;dx$ by finding the limit of a sequence of Riemann sums.

(b) Evaluate $\int_0^1 e^x\,dx$ by finding the limit of a sequence of Riemann sums.

4: (a) Find $g'(1)$ where $g(x)=\int_{3x-3}^{x^2+1}\sqrt{1+t^3}\,dt$.

(b) Find $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{1}{n+k}$.

Hints and Comments

1: The graph of $y = g(x)$ can be found by interpreting the integral as a signed area. Alternatively, you can find an explicit formula for $g(x)$ by first obtaining an explicit piecewise formula for $f(x)$, which you can determine from its graph.

2: (b) We remark that, by the FTC, the exact value of the given integral is

$$\int_{1/5}^{13/5} \frac{dx}{x} = \Big[\ln x\Big]_{1/5}^{13/5} = \ln \tfrac{13}{5} - \ln \tfrac{1}{5} = \ln 13\,,$$

and so the approximation of the integral by Riemann sums can be thought of as a numerical approximation of the value $\ln 13$.

3: (a) You will need the formulas

$$\sum_{k=1}^{n} 1 = n\ ,\ \sum_{k=1}^{n} k = \frac{n(n+1)}{2}\ ,\ \sum_{k=1}^{n} k^2 = \frac{n\big(n+\frac{1}{2}\big)\big(n+1\big)}{3}\ ,\ \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}\,.$$

(b) The Riemann sum that you obtain should be geometric. Recall that the sum of a geometric series is given by the formula

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}\,.$$

To find the limit of the Riemann sums, you can make use of l'Hôpital's Rule (strictly speaking, in order to use l'Hôpital's Rule, you should first replace the discreet variable n by a continuous variable x).

4: (a) Let $u(x) = x^2 + 1$ and $v(x) = 3x - 3$, and let $f(t) = \sqrt{1+t^3}$ and $F(u) = \displaystyle\int_0^u f(t)\,dt$. Show that we have $g(x) = F\big(u(x)\big) - F\big(v(x)\big)$ then use the Chain Rule and the FTC to find $g'(x)$.

(b) Find a function $f(x)$ and an interval $[a,b]$ such that the given sum is equal to a Riemann sum for $f(x)$ on $[a,b]$.