MATH 138 Calculus 2, Assignment 5 Due Mon Nov 18, 11:00 pm

1: (a) Verify that $y = x\sin x$ is a solution of the DE $y\,(y'' + y) = x\sin 2x$.

(b) Find all the solutions of the form $y = ax^2 + bx + c$ to the DE $\big(y'(x)\big)^2 + 4x = 3y(x) + x^2 + 1$.

(c) Find constants r_1 and r_2 such that $y = e^{r_1 x}$ and $e^{r_2 x}$ are both solutions to the DE $y'' + 3y' + 2y = 0$, show that $y = a\,e^{r_1 x} + b\,e^{r_2 x}$ is a solution for any constants a and b, and then find a solution to the DE with $y(0) = 1$ and $y'(0) = 0$.

2: Find the general solution to each of the following DEs.

(a) $x\,y' + y = \sqrt{x}$

(b) $\sqrt{x}\,y' = 1 + y^2$

(c) $y' = 2xy^2 + y^2 + 8x + 4$

(d) $y' + y\tan x = \sin^2 x$

3: Find the solution to each of the following IVPs.

(a) $x\,y' = y^2 + y$ with $y(1) = 1$.

(b) $x\,y' + 2y = \ln x$ with $y(1) = 0$.

(c) $y' + xy = x^3$ with $y(0) = 1$.

4: Solve the initial value problem $y'' - 2y' = 4x$ with $y(0) = 0$ and $y'(0) = 0$.
(Hint: first let $u(x) = y'(x)$ so that $y''(x) = u'(x)$ and then solve the resulting DE for $u = u(x)$).

(b) Solve the IVP $y\,y'' + (y')^2 = 0$ with $y(1) = 2$ and $y'(1) = 3$.
(Hint: first let $u(y(x)) = y'(x)$ so that $u'(y(x))y'(x) = y''(x)$ and then solve the resulting DE for $u = u(y)$).