MATH 138 Calculus 2, Assignment 6 Not to hand in

1: (a) Find the length of the curve $y = \sqrt{4x - x^2}$ with $0 \le x \le 3$.

(b) Find the length of the curve $y = 3\,x^{2/3}$ with $0 \le x \le 8$.

2: (a) Find the area of the surface obtained by revolving the curve $y = \sqrt{x}$ with $0 \le x \le 2$ about the x-axis.

(b) Find the area of the surface which is obtained by revolving the curve $y = \cos x$ with $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ about the x-axis.

3: Consider the IVP $y' = 2(x + y) - \frac{1}{2}$ with $y(0) = 0$.

(a) Find the exact solution $y = f(x)$ to the above IVP.

(b) Apply Euler's method with step size $\Delta x = \frac{1}{2}$ to find a polygonal approximation $y = g(x)$ for $0 \le x \le 2$ to the above solution $y = f(x)$.

(c) Sketch the direction field for the given DE along with the graph of the exact solution $y = f(x)$ and the graph of the polygonal solution $y = g(x)$.

4: (a) The amount $A(t)$ of a radioactive substance satisfies the DE

$$A'(t) = k\,A(t)$$

for some constant $k < 0$. The substance has a half-life of 10 seconds, which means that $A(10) = \frac{1}{2}\,A(0)$. If $A(5) = 100$ then find the exact time t at which $A(t) = 20$.

(b) A murder victim is found in a room of constant teperature 25° C. At the time of murder, we assume that the victim's body temperature was 37° C. At 2:00 pm, the body temperature is measured to be 31° C and at 5:00 pm, it is measured to be 29° C. Determine the time of death, assuming that the temperature $T = T(t)$ of the body at time t satisfies Newton's Law of Cooling $T' = -k(T - 25)$ for some constant $k > 0$.

5: A tank initially contains 20 L of pure water. Brine containing 5 grams of salt per liter of water enters the tank at 6 L/min. The solution is kept well mixed and drains from the tank at 2 L/min. Find the concentration of salt in the tank when the tank contains 80 L of brine.