MATH 138 Calculus 2, Assignment 7 Due Mon Dec 2, 11:00 pm

1: Find the sum of each of the following series, if the sum exists.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1}-1}{3^n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{4^{1-n}}{3^{n-1}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2+3n}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{(n+1)!}$

2: Determine which of the following series converge.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}}{2n^2+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\, 2^{1/n}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{3^n n!}$

3: (a) Approximate the sum $S = \displaystyle\sum_{n=0}^{\infty} \frac{1}{5^n+5n}$ by a partial sum S_ℓ so that the error is $S - S_\ell \le \frac{1}{500}$.

(b) Let $S = \displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$. Use a calculator to find a value of ℓ such that $S - S_\ell \le \frac{1}{500}$.

(c) Let $f(x) = \dfrac{1}{x(\ln x)^2}$, let $a_n = f(n)$ for $n \ge 2$, let $S = \displaystyle\sum_{n=2}^{\infty} a_n$, and let $S_\ell = \displaystyle\sum_{n=2}^{\ell} a_n$. Find a value of ℓ such that if we approximate S by $S \cong S_\ell$ then the absolute error is $|S - S_\ell| \le \frac{1}{100}$ and, using a calculator, find another value of ℓ such that if we approximate S by

$$S \cong T_\ell = S_\ell + \tfrac{1}{2}\left(\int_\ell^\infty f(x)\,dx + \int_{\ell+1}^\infty f(x)\,dx\right)$$

then the absolute error is $|S - T_\ell| \le \frac{1}{100}$.

4: Determine, with proof, which of the following statements are true for all sequences $(a_n)_{n\ge1}$ and $(b_n)_{n\ge1}$.

(a) If $\sum a_n$ converges that $\sum e^{a_n}$ diverges.

(b) If $\sum a_n$ converges then $\sum {a_n}^2$ converges.

(c) If $a_n \ge 0$ for all n and $\sum a_n$ converges then $\displaystyle\sum \frac{a_n}{1+a_n}$ converges.

(d) If $\sum a_n$ converges and $\sum |b_n|$ converges then $\sum a_n b_n$ converges.