MATH 138 Calculus 2, Exercises for Appendix 1

1: (a) Find the exact value of $\cos\left(\frac{\pi}{5}\right)$. Express your answer in terms of integers and radicals.
Hint: let $\theta = \frac{\pi}{5}$ and consider a triangle with angles θ, 2θ and 2θ cut into two triangles, one with angles θ, θ and 3θ, and the other with angles θ, 2θ and 2θ.

(b) Find the area of a regular decagon with sides of length 1 (a decagon has 10 sides).

2: Let A be the rectangle-based cone with its base vertices at $(\pm 2, \pm 1, 0)$ and with its top vertex at $(0, 3, 4)$, and let B be the rectangle-based cone with the same base but with its top vertex at $(0, -3, 4)$. Find the volume and the surface area of the solid $A \cup B$.

3: Let R be the radius of the Earth ($R \cong 6,000$ km).

(a) A satellite orbits the Earth at a distance $2R$ from the Earth's center. Let A be the set of points on the Earth's surface from which the satellite is visible (at some instant in time). Find the area of A.

(b) Let B be the portion of the Earth's surface which lies between 30° and 60° latitude and between 30° and 60° longitude. Find the area of B.

4: (a) Let A be the ball of radius 2 centered at $(1, 0, 0)$ and let B be the ball of radius 2 centered at $(-1, 0, 0)$. Find the volume of the solid $A \cap B$.

(b) A cylindrical hole is bored through the centre of a solid spherical ball. Let A be the portion of the ball which remains. Let h be the height of the cylindrical face of A. Find the volume of A in terms of h (somewhat surprisingly, the final answer involves neither the radius of the sphere, nor the radius of the hole).

5: (a) Let A be the the solid torus obtained by revolving the disc $(x - R)^2 + y^2 \leq r^2$ about the y-axis. Find the volume and the surface area of A. (Hint: slice A into pieces which can be reassembled to form a cylinder).

(b) Let B be the paraboloidal solid which is obtained by revolving the region given by $0 \leq x \leq 1$ and $x^2 \leq y \leq 1$ about the y-axis. Find the volume of B. (Hint: slice B horizontally into n thin discs each of thickness $\frac{1}{n}$, find the approximate volume of each disc by treating it as a cylinder, add these volumes and take the limit as $n \to \infty$).