MATH 138 Calculus 2, Exercises for Chapter 3

1: A tank initially contains 500 L of water. Water drains out at a rate $r(t)$ L/min. Some values of $r(t)$ are tabulated below.

t	0	10	20	30	40	50	60
r	1.0	1.4	2.0	2.8	4.0	5.8	8.0

(a) Estimate the amount of water remaining in the tank after one hour by approximating the definite integral $\int_0^{60} r(t)\,dt$ using the midpoint rule on 3 subintervals.

(b) Estimate the amount of water remaining in the tank after one hour by approximating the same integral using the Trapezoidal Rule on 6 subintervals.

2: Suppose that $\left|f''(x)\right| \leq \frac{1}{2}$ and $\left|f''''(x)\right| \leq 2$ for all $x \in [0, 24]$, and that $f(x)$ has the following table of values.

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
$f(x)$	4	5	4	3	2	2	2	3	4	5	6	3	2	1	1	2	4	4	4	3	71	1	3	4	7

(a) Use S_8 to approximate $\int_0^{24} f(x)\,dx$.

(b) Find a value of n such that if we estimate $\int_0^{24} f(x)\,dx$ using T_n the error is $E \leq .01$.

3: (a) Approximate $\int_0^{2\pi} 4^{\cos x}\,dx$ using R_6.

(b) Find a value of n such that if we approximate $\int_0^4 3\sqrt{2x+1}\,dx$ using S_n then the error is $E \leq .0001$.

4: Evaluate the following improper integrals.

(a) $\int_0^2 x^3 \ln(x/2)\,dx$

(b) $F(s) = \int_0^\infty e^{-st} \sin t\,dt$, where $s > 0$.

5: Evaluate the following improper integrals.

(a) $\int_2^\infty \frac{dx}{x^4\sqrt{x^2-4}}$

(b) $\int_{-\infty}^\infty \frac{x(x+1)}{(x^2+1)^2}\,dx$