MATH 138 Calculus 2, Exercises for Chapter 5

1: Find a solution of the form $y = a + bx + cx^2$ to the differential equation $y'' + (1+x)y' - 2y = 5x - 3$ with $y(1) = 4$.

2: Solve the following differential equations.

(a) $x\,y' + 2y = \sqrt{1+x^2}$

(b) $y' = e^{x+2y}$

3: Solve the following initial value problems.

(a) $(1+x^2)y' = x\,y$ with $y(0) = 2$.

(b) $x^2 y' - y = 1$ with $y(1) = 1$.

4: (a) Solve the initial value problem $y'' - 2y' = x$ with $y(0) = 0$ and $y'(0) = 0$.
(Hint: first let $u(x) = y'(x)$ so that $y''(x) = u'(x)$ and solve the resulting DE for $u = u(x)$).

(b) Solve the initial value problem $y'' = (1-2y)\,y'$ with $y(0) = 1$ and $y'(0) = 2$.
(Hint: first let $u(y(x)) = y'(x)$ so that $u'(y(x))y'(x) = y''(x)$ and solve the resulting DE for $u = u(y)$).

5: The amount $A(t)$ of a radioactive substance decays exponentially with a half-life of 3 seconds. If $A(2) = 20$ then find the time t at which $A(t) = 4$.

6: A pot of boiling water is removed from the heat and placed on a table in a room. After 2 minutes, the water has cooled from $100°$ to $84°$. After another 2 minutes, it has cooled to $72°$. What is the temperature in the room?

7: Water drains from a hole of area 25 cm^2 at the bottom tip of a conical tank of radius 1 m and height 4 m. If the water drains at a velocity of $v = 4\sqrt{y}\ \ m/s$, where y m is the depth of the water in the tank, then find the time at which the tank will be empty.

8: A tank contains 100 L of water. A solution with salt concentration 0.5 kg/L is added at 6 L/min. The solution is kept well mixed and is drained from the tank at a rate of 4 L/min. Find the concentration of salt in the tank when it contains 200 L of solution.

9: Let $x(t)$ be the height of an object of mass m which is thrown upwards from the ground. If the force of air resistance is $-kx'$, then $x(t)$ satisfies the DE $mx'' + kx' + mg = 0$. Suppose that $m = 1$, $k = \frac{1}{10}$, $g = 10$, $x(0) = 0$ and $x'(0) = 20$. Find the time t at which the object reaches its maximum height.