MATH 138 Calculus 2, Exercises for Chapter 7

1: Find the limit of each of the following sequences (a_n), if the limit exists.

(b) $a_n = \dfrac{(-3)^n}{2^{2n+1}}$ (c) $a_n = \dfrac{2^{2n}}{n!}$ (d) $a_n = \left(\dfrac{n+1}{n-1}\right)^n$.

2: (a) Let $a_1 = \frac{4}{3}$ and $a_{n+1} = 5 - \dfrac{4}{a_n}$ for $n \geq 1$. Determine whether (a_n) converges and, if so, find the limit.

(b) Let $a_1 = 2$ and $a_{n+1} = \sqrt{3{a_n}^2 - 3}$ for $n \geq 1$. Determine whether (a_n) converges and, if so, find the limit.

3: (a) Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{2^{2n+1}}$, if it exists.

(b) Evaluate $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+4n+3}$, if it exists.

(c) A hypothetical ball bounces as follows: when it is in the air, it has a constant downwards acceleration of $g = 10$; when it bounces, it rebounds instantaneously; whenever it drops from a height h, it rebounds to a height of $\frac{3}{4}h$. This ball is dropped from an initial height $h = 5$ and allowed to bounce indefinitely. Find the total distance travelled by the ball, and determine how long it takes for the ball to come to rest.

4: Determine which of the following series converge.

(a) $\displaystyle\sum \frac{n^2+4n}{\sqrt{n^5-2n+1}}$ (b) $\displaystyle\sum \frac{n^4}{2^n}$ (c) $\displaystyle\sum \frac{1}{n\,(\ln n)^2}$ (d) $\displaystyle\sum \frac{n^n}{n!}$ (e) $\displaystyle\sum \frac{\ln n}{\sqrt{n}}$

5: For each of the following series, determine whether it converges absolutely, converges conditionally, or diverges.

(a) $\displaystyle\sum \frac{(-1)^n\sqrt{n}}{n+2}$ (b) $\displaystyle\sum (-1)^n e^{1/n}$ (c) $\displaystyle\sum \frac{(-1)^n}{\ln n}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$ (e) $\displaystyle\sum \frac{n}{(-2)^n}$

6: (a) Approximate the sum $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n+2}$ so that the absolute error is at most $\frac{1}{30}$.

(b) Approximate the sum $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+n}$ so that the absolute error is at most $\frac{1}{16}$.

(c) Approximate the sum $\displaystyle\sum_{n=2}^{\infty} \frac{n-1}{n!}$ so that the absolute error is at most $\frac{1}{100}$.

7: Determine, with proof, which of the following statements are true for all sequences $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$.

(a) If $\sum a_n$ converges then $\sum \cos(a_n)$ diverges.

(b) If $a_n \geq 0$ for all n and $\sum a_n$ converges then $\sum {a_n}^2$ converges.

(c) If $b_n \neq 0$ for all n and $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 1$ then $\left(\sum a_n \text{ converges} \iff \sum b_n \text{ converges}\right)$.