MATH 138 Calculus 2, Exercises for Chapter 8

1: (a) Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty}\frac{(1-4x)^n}{n\,2^n}$.

(b) Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty}\frac{(2-3x)^n}{n+\sqrt{n}}$.

(c) Find the set of all values of x such that the series $\displaystyle\sum_{n=1}^{\infty}\frac{(x^2+x-1)^n}{n}$ converges.

2: (a) Find the Taylor Polynomial of degree 5 centred at 0 for $f(x)=\dfrac{e^x}{1+x}$.

(b) Find the Taylor Polynomial of degree 5 centred at 0 for $f(x)=(1+2x)^{3/2}\sin x$.

(c) Find the Taylor polynomial of degree 4, centred at 0, for $f(x)=\dfrac{\ln(1+x)}{\tan^{-1}x}$.

3: (a) Find the Taylor series centred at 0 for $f(x)=\dfrac{-4}{x^2+2x-3}$, and find the radius of convergence.

(b) Find the Taylor series centred at -1 for $f(x)=\dfrac{-4}{x^2+2x-3}$, and find the radius of convergence.

(c) Find the Taylor series centred at 0 for $f(x)=\sin x\cos x$.

(d) Find the Taylor series centred at $\frac{\pi}{4}$ for $f(x)=\sin x\cos x$.

4: (a) Approximate $(1300)^{2/3}$ so that the absolute error is at most $\frac{1}{200}$.

(b) Approximate $\ln(4/5)$ so that the absolute error is at most $\frac{1}{100}$.

(c) Approximate $\displaystyle\int_0^{1/5}\frac{\ln(1+x)}{x}\,dx$ so that the absolute error is at most $\frac{1}{1000}$.

5: (a) Let $f(x)=\cos^2\left(\dfrac{x^2}{4\sqrt{3}}\right)$. Find the twelfth derivative $f^{(12)}(0)$.

(b) Evaluate $\displaystyle\lim_{x\to 0}\frac{\sin x\tan^{-1}x-x^2}{\cos(x^2)-1}$.

(c) Evaluate $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^n}{(2n)!}$.

(d) Evaluate $\displaystyle\sum_{n=0}^{\infty}\frac{n}{(n+1)!}$.

6: (a) Let $c_n=1$ when n is even and $c_n=2$ when n is odd. Find the function $f(x)$ whose Taylor series centred at 0 is equal to $\sum_{n=0}^{\infty}c_nx^n$.

(b) Let $f(x)=x^3+x+1$. Note that $f(x)$ is increasing with $f(0)=1$. Let $g(x)=f^{-1}(x)$, Find the Taylor polynomial of degree 6 centred at 1 for $g(x)$.

(c) Find the Taylor polynomial of degree 5 centred at 0 for the solution to the IVP $\frac{1}{2}y''+y'-3y=x+1$ with $y(0)=1$ and $y'(0)=2$.