MATH 247 Calculus 3, Assignment 1 Due Mon Sept 30, 11:00 pm

1: Consider the surface given implicitly by $x^2 + y^2 + 1 = yz$.

(a) Sketch the level sets $z = \pm 2, \pm 4, \pm 6$ and the level set $x = 0$ for this surface.

(b) Sketch the surface.

(c) Find an explicit equation for the tangent plane to the surface at the point $(2, 1, 6)$.

2: (a) Sketch the curve given parametrically by $(x, y) = \left(\frac{2}{1+t^2}, \frac{2t}{1+t^2}\right)$, showing all points at which the tangent line is horizontal or vertical, then find an implicit equation for the curve.

(b) Define $f : \mathbb{R} \to \mathbb{R}^2$ by $f(t) = \left(t^2 \, , \, \frac{t}{t^2+1}\right)$ and define $g : \mathbb{R}^2 \to \mathbb{R}$ by $g(x, y) = y^2(x + 1)^2 - x$.
Prove that $\text{Range}(f) = \text{Null}(g)$, then find an explicit equation for the tangent line to this curve at $\left(\frac{1}{4}, \frac{2}{5}\right)$.

3: (a) Find a parametric equation for the tangent line to the curve of intersection of the paraboloid $z = 1-x^2-y^2$ with the plane $z = 1 - 2x$ at the point $(1, 1, -1)$.

(b) When we consider the function $f : \mathbb{C} \to \mathbb{C}$ given by $f(z) = z^2$ as a function $f : \mathbb{R}^2 \to \mathbb{R}^2$, it is given by $f(x, y) = \big(u(x, y), v(x, y)\big)$ with $u(x, y) = x^2-y^2$ and $v(x, y) = 2xy$. Let $A = \left\{(x, y) \in \mathbb{R}^2 \,\middle|\, 1 \le x \le 2, 0 \le y \le x\right\}$ and $B = \left\{(u, v) \in \mathbb{R}^2 \,\middle|\, 1 \le u \le 4 \, , \, 0 \le v \le 2\right\}$. Accurately sketch or describe the sets $f(A)$ and $f^{-1}(B)$.

4: (a) Find an implicit equation, of the form $ax + by + cz = d$, for the tangent plane to the parametric surface $(x, y, z) = f(s, t) = \left(s - t^2 \, , \, \frac{s}{t} \, , \, \sqrt{s\,t}\right)$ at the point where $(s, t) = (4, 1)$.

(b) Let C be the set of all $(u, v, w) \in \mathbb{R}^3$ such that the polynomial $f(x) = x^3 + ux^2 + vx + w$ has a triple real root, and let S be the set of all $(u, v, w) \in \mathbb{R}^3$ such that the polynomial. $f(x) = x^3 + ux^2 + vx + w$ has a multiple real root (that is a double or triple real root). Find a parametric equation for C and find a parametric equation and an implicit equation for S. As an optional additional exercise (not to be marked), use computer software to display the curve C and the surface S.