MATH 247 Calculus 3, Assignment 2 Not to hand in

1: Read Theorem 4.20 (the Chain Rule) and Definition 4.24 (the directional derivative), then solve the following.

(a) Let $(u,v) = g(z) = \big(\sqrt{z-1}, 5\ln z\big)$, where $z = f(x,y) = 4x^2 - 8xy + 5y^2$. Use the Chain Rule to find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ at the point $(2,1)$ (this is Exercise 4.21).

(b) Let $(x,y) = f(r,\theta) = \big(r\cos\theta\,,\, r\sin\theta\big)$, let $z = g(x,y)$ and let $z = h(r,\theta) = g\big(f(r,\theta)\big)$. Suppose that $h(r,\theta) = r^2 e^{\sqrt{3}(\theta - \frac{\pi}{6})}$. Find $Dg(\sqrt{3},1)$ (this is Exercise 4.22).

(c) Let $f(x,y,z) = x\,\sin(y^2 - 2xz)$ and let $\alpha(t) = \big(\sqrt{t}\,,\, \frac{1}{2}t\,,\, e^{(t-4)/4}\big)$. Find the rate of change of f as we move along the curve $\alpha(t)$ when $t = 4$ (this is Exercise 4.25).

2: Read Theorem 4.26 then solve the following.

(a) Consider the surface $z = f(x,y)$ where $f(x,y) = \dfrac{4}{2 + x^4 + x^2 + y^2}$. An ant walks counterclockwise around the curve of intersection of the above surface $z = f(x,y)$ with the cylinder $(x-2)^2 + y^2 = 5$. Find the value of $\tan\theta$, where θ is the angle (from the horizontal) at which the ant is ascending when it is at the point $\big(1, 2, \frac{1}{2}\big)$.

(b) Consider the surface $z = f(x,y)$ where $f(x,y) = \dfrac{6x}{1 + x^2 + y^2}$. Show that any circle which passes through the points $(1,0)$ and $(-1,0)$ is a curve of steepest descent, that is for any point (x,y) on any circle C through the two points $(-1,0)$ and $(1,0)$, the slope of C at (x,y) is equal to the slope of the gradient vector $\nabla f(x,y)$.

3: (a) Find $\displaystyle\iint_D y\,e^x\,dA$ where D is the region in $\mathbb{R}^2$ bounded by $y = 0$, $y = x$ and $x + y = 2$.

(b) Find $\displaystyle\iint_D \frac{x}{\sqrt{1 + x^2 + y^2}}\,dA$ where $D = \big\{(x,y)\,\big|\,0 \le x \le 2\,,\ 0 \le y \le \frac{1}{2}x^2\big\}$.

(c) Find $\displaystyle\iiint_D z\ dV$ where $D = \big\{(x,y,z)\,\big|\,0 \le x\,,\ 0 \le y \le \sqrt{x^2 + z^2}\,,\ 0 \le z \le \sqrt{1 - x^2}\big\}$.

4: (a) Find $\displaystyle\iint_D \cos(3x^2 + y^2)\,dA$ where $D = \big\{(x,y)\,\big|\,x^2 + \frac{1}{3}\,y^2 \le 1\big\}$.

(b) Find $\displaystyle\iint_D e^{(y-x)/(y+x)}\,dA$ where D is the quadrilateral with vertices at $(1,1)$, $(2,0)$, $(4,0)$, $(2,2)$.

(c) Find $\displaystyle\iiint_D (x-y)z\,dV$ where $D = \big\{(x,y,z)\,\big|\,x^2 + y^2 + z^2 \le 4\,,\ z \ge \sqrt{x^2 + y^2}\,,\ x \ge 0\big\}$.

5: Read Note 7.23, then solve the following.

(a) Find the total charge in the region $D = \left\{(x,y,z)\middle|\sqrt{\frac{1}{3}(x^2 + y^2)} \le z \le \sqrt{4 - x^2 - y^2}\right\}$ where the charge density (charge per unit volume) is given by $f(x,y,z) = x^2$.

(b) Find the mass of the sphere $x^2 + y^2 + z^2 = 1$ when the density (mass per unit area) is given by $f(x,y,z) = 3 - z$ (This is Exercise 7.26).

(c) Find the mass of the curve of intersection of the parabolic sheet $z = x^2$ with the paraboloid $z = 2 - x^2 - 2y^2$ when the density (mass per unit length) is given by $f(x,y,z) = |xy|$ (This is Exercise 7.27).