MATH 247 Calculus 3, Assignment 4 Due Mon Nov 4, 11:00 pm

1: (a) Let $A, B \subseteq \mathbb{R}^n$. Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(b) Let $A \subseteq \mathbb{R}^n$. Show that $A' = \overline{A}'$ or, in other words, show that A and $\overline{A}$ have the same limit points.

(c) Let $A, B \subseteq \mathbb{R}^n$ be disjoint closed sets. Show that there exist disjoint open sets $U, V \subseteq \mathbb{R}^n$ with $A \subseteq U$ and $B \subseteq V$.

2: (a) Let $A, B \subseteq \mathbb{R}^n$. Show that $\partial(A \cup B) \subseteq \partial A \cup \partial B$.

(b) Let $A, B \subseteq \mathbb{R}^n$. Show that $\partial(A \cap B) \subseteq (A \cap \partial B) \cup (B \cap \partial A) \cup (\partial A \cap \partial B)$.

(c) Give an example of sets $A, B \subseteq \mathbb{R}$ for which $\partial(A \cap B) \neq (A \cap \partial B) \cup (B \cap \partial A) \cup (\partial A \cap \partial B)$.

3: (a) Let $a \in \mathbb{R}^n$, let $r > 0$, and let $B(a, r) \subseteq A \subseteq \overline{B}(a, r)$. Show that $A^o = B(a, r)$ and $\overline{A} = \overline{B}(a, r)$.

(b) Determine whether for every subset $P \subseteq \mathbb{R}^n$, we have $\overline{B_P(a, r)} = \overline{B}_P(a, R)$ for all $a \in P$ and all $r > 0$.

(c) Let $A \subseteq P \subseteq \mathbb{R}^n$. Prove that A is compact in P if and only if A is compact in $\mathbb{R}^n$.

4: (a) Let $A \subseteq \mathbb{R}^n$ be compact and let S be an open cover of A. Show that there exists $r > 0$ such that for every $a \in A$ there exists $U \in S$ such that $B(a, r) \subseteq U$.

(b) Let $C_1, C_2, C_3, \cdots$ be non-empty closed sets in $\mathbb{R}^n$ with $C_1 \supseteq C_2 \supseteq C_3 \supseteq \cdots$. Show that if each set C_k is compact then $\bigcap\limits_{k=1}^{\infty} C_k \neq \emptyset$, and find an example where the sets C_k are not compact and we have $\bigcap\limits_{k=1}^{\infty} C_k = \emptyset$.