MATH 247 Calculus 3, Exercises for Chapter 1

1: Let $A = \text{Range}(f)$ where $f : \mathbf{R} \to \mathbf{R}^2$ is given by $f(t) = \big(\cos t, \sin 2t\big)$ and let $B = \text{Null}(g)$ where $g : \mathbf{R}^2 \to \mathbf{R}$ is given by $g(x, y) = y^2 + 4x^2(x^2 - 1)$. Show (algebraically) that $A = B$, and then sketch the set $A \subseteq \mathbf{R}^2$ (it is a curve in $\mathbf{R}^2$).

2: A light, represented by the point $(0, 0, 5)$, lies above the ground, which is represented by the xy-plane. The position of a fly at time $t \geq 0$ is given by $(x, y, z) = (t, t^2, t^3)$. Find the position of the shadow of the fly at time t (you are finding a parametric equation for the curve in the xy-plane traced by the shadow of the fly).

3: Let $f(x, y) = 2^{y-x^2}$. Sketch the level sets $z = \frac{1}{4}, \frac{1}{2}, 1, 2, 4$ and the level sets $x = 0$ and $y = 0$, and then sketch the surface $z = f(x, y)$ (the graph of f).

4: Let $f(x, y, z) = 4x^2 + y^2 - yz$. Sketch the level sets $z = 0, \pm 1, \pm 2, \pm 3, \pm 4$ and the level sets $x = 0$ and $y = 0$, and then sketch the surface $f(x, y, z) = 0$ (the null set of f).

5: Let $f(x, y) = x^2 + 2y^2$ and $g(x, y) = 4x - y^2$. Find a parametric equation for the curve of intersection of the two surfaces $z = f(x, y)$ and $z = g(x, y)$.