MATH 247 Calculus 3, Exercises for Chapter 2

1: Let $0 \neq u, v, w \in \mathbf{R}^n$.

(a) (Trigonometric Ratios) Show that if $(v-u) \cdot u = 0$ then $\cos\theta(u,v) = \frac{|u|}{|v|}$ and $\sin\theta(u,v) = \frac{|v-u|}{|v|}$

(b) (Angle Addition) Show that if $0 \neq w = su + tv$ for some $s, t \geq 0$ then we have $\theta(u,v) = \theta(u,w) + \theta(w,v)$.

2: (a) Let $A = \{(x,y) \in \mathbf{R}^2 \,|\, 0 < x,\ 0 < y \text{ and } xy < 1\}$. Show, from the definition of an open set, that A is open in $\mathbf{R}^2$.

(b) Let $B = \left\{ \left(\frac{2t}{t^2+1}, \frac{t^2-1}{t^2+1}\right) \in \mathbf{R}^2 \,\middle|\, t \in \mathbf{R} \right\}$. Show that B is not closed in $\mathbf{R}^2$.

3: Let $A \subseteq \mathbf{R}^n$.

(a) Show that A' is closed in $\mathbf{R}^n$.

(b) Show that $\partial A = \overline{A} \setminus A^o$.

4: (a) Let $A, B \subseteq \mathbf{R}^n$ show that if A is connected and $A \subseteq B \subseteq \overline{A}$ then B is connected.

(b) Let S be a nonempty set and let $A_j \subseteq \mathbf{R}^n$ for each $j \in S$. Suppose that A_j is connected for all $j \in S$ and that $A_k \cap A_\ell \neq \emptyset$ for all $k, \ell \in S$. Show that $\bigcup_{j \in S} A_j$ is connected.

5: Let $A \subseteq P \subseteq \mathbf{R}^n$. Define the **interior of** A **in** P to be the union of all sets $E \subseteq P$ such that E is open in P and $E \subseteq A$. Define the **closure of** A **in** P to be the intersection of all sets $F \subseteq P$ such that F is closed in P and $A \subseteq F$. Denote the interior of A in $\mathbf{R}^n$ and the closure of A in $\mathbf{R}^n$ by A^o and $\overline{A}$ (as usual). Denote the interior of A in P and the closure of A in P by $\text{Int}_P(A)$ and $\text{Cl}_P(A)$.

(a) Show that $\text{Cl}_P(A) = \overline{A} \cap P$.

(b) Show that $\text{Int}_P(A) = (A \cup P^c)^o \cap P$, where $P^c = \mathbf{R}^n \setminus P$.