MATH 247 Calculus 3, Exercises for Chapter 4

1: (a) Find an implicit and an explicit equation for the tangent line to the parametric curve $(x, y) = (\cos t, \sin 2t)$ at the point where $t = \frac{\pi}{3}$.

(b) The position of a fly at time t is given by $(x, y, z) = (t, t^2, 1 + t^3)$. A light shines down on the fly from the point $(0, 0, 3)$ and casts a shadow on the xy-plane. Find the position and the velocity of the shadow of the fly at time $t = 1$.

2: Let S be the parametric surface $(x, y, z) = f(s, t) = \left(\frac{s}{t}, \sqrt{s + t}, st\right)$.

(a) Find the derivative matrix $Df(s, t)$.

(b) Find a parametric equation for the tangent plane to S at the point where $(s, t) = (2, 2)$.

(c) Find an implicit equation for the tangent plane to S at the point where $(s, t) = (2, 2)$.

3: Let C be the curve of intersection of the two surfaces $z = x^2 - 2y$ and $z = 2x^2 + y^2$. Find a parametric equation for the tangent line L to the curve C at the point $(-1, -1, 3)$ using each of the following two methods.

(a) Find the equation of the tangent plane to each of the two surfaces at $(-1, -1, 3)$, then solve the two equations to obtain a parametric equation for L.

(b) Find a parametric equation for C, then use this parametric equation to find a parametric equation for the tangent line L.

4: (a) Let P be the tangent plane to the surface given by $z = 4x^2 - 8xy + 5y^2$ at the point where $(x, y) = (2, 1)$. Find the line of intersection of P with the xy-plane.

(b) Find the equation of the tangent plane to the surface given by $e^{x+z} = \sqrt{x^2 y + z}$ at the point $(1, 2, -1)$.

5: Let S be the surface $2yz = x^2 + y^2$.

(a) Sketch the level sets $z = -2, -1, 0, 1, 2$ for the surface S (in other words, sketch the curve of intersection of S with the each of the planes $z = -2, -1, 0, 1, 2$).

(b) Sketch the surface S.

(c) Find the equation of the tangent plane to S at the point $(3, 1, 5)$.